Accelerated Precalculus
4.14 Quiz Review

$\ast\ 30, 45, 60, 90°$

Name _____

Do not use a calculator!
Find the exact values using angle sum or difference identities.

1. $\sin 105°$ $\quad \sin(45+60)$

$\ast\ \sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(45+60) = \sin 45 \cos 60 + \cos 45 \sin 60$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{6}}{4}$

$\boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

2. $\tan\left(\frac{17\pi}{12}\right)$ $\quad \dfrac{3\pi}{12} + \dfrac{14\pi}{12} = \dfrac{17\pi}{12}$

$\dfrac{\pi}{4} + \dfrac{7\pi}{6}$

$\tan\left(\pi/4 + 7\pi/6\right)$

$\dfrac{\tan(\pi/4) + \tan(7\pi/6)}{1 - \tan(\pi/4)\tan(7\pi/6)}$

$\dfrac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)} = \boxed{\sqrt{3}+2}$

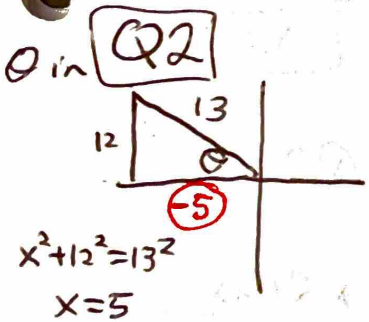3. $\cos 1005°$ $\quad \cos(285)$

$\cos(60 + 225)$

$\ast\ \cos(A+B) = \cos A \cos B - \sin A \sin B$

$\rightarrow \cos 60 \cos 225 - \sin 60 \sin 225$

$\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$-\dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{6}}{4} = \boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

4. If $\sin\theta = \frac{12}{13}$ and $\cos\theta < 0$, find $\tan 2\theta$.

θ in $\boxed{Q2}$

$x^2 + 12^2 = 13^2$
$x = 5$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$
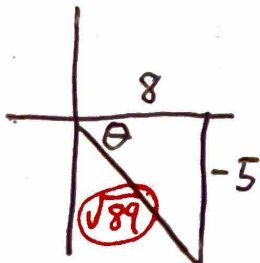
$\tan 2\theta = \dfrac{2\left(\frac{-12}{5}\right)}{1 - \left(\frac{-12}{5}\right)^2}$

$\tan 2\theta = \dfrac{\frac{-24}{5}}{1 - \frac{144}{25}} \rightarrow$

$\boxed{\tan 2\theta = \dfrac{120}{119}}$

5. If $\tan\theta = \frac{-5}{8}$ and $\sec\theta > 0$, find $\cos 2\theta$.

θ is in Q4

$8^2 + (-5)^2 = c^2 \quad \big| \quad c^2 = 89$
$64 + 25 = c^2 \quad \big| \quad c = \sqrt{89}$

$\cos 2\theta = 2\cos^2\theta - 1$

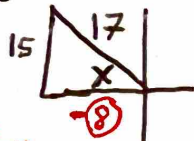$= 2\left(\frac{8}{\sqrt{89}}\right)^2 - 1$

$= 2\left(\frac{64}{89}\right) - 1 = \boxed{\dfrac{39}{89}}$

$= \dfrac{128}{89} - \dfrac{89}{89}$
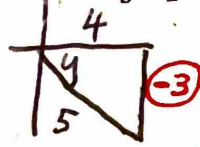
$$a^2 + 15^2 = 17^2$$
$$a = 8$$

6. Find the exact value of each given that $\sin x = \frac{15}{17}$, $\frac{\pi}{2} < x < \pi$ **and** $\cos y = \frac{4}{5}$, $\frac{3\pi}{2} < y < 2\pi$.

$$\sin x = \frac{15}{17} \quad \cos x = \frac{-8}{17}$$

Q2

$$\cos y = 4/5$$

Q4

$$\sin y = \frac{-3}{5}$$

$* \cos(x+y) = \cos x \cos y - \sin x \sin y$

a. $\cos(x + y)$

$$\rightarrow \left(\frac{-8}{17}\right)\left(\frac{4}{5}\right) - \left(\frac{15}{17}\right)\left(\frac{-3}{5}\right)$$

$$\rightarrow -\frac{32}{85} + \frac{45}{85} = \boxed{\frac{13}{85}}$$

$* \sin(y-x) = \sin y \cos x - \cos y \sin x$

b. $\sin(y - x)$

$$\left(\frac{-3}{5}\right)\left(\frac{-8}{17}\right) - \left(\frac{4}{5}\right)\left(\frac{15}{17}\right)$$

$$= \frac{24}{85} - \frac{60}{85} = \boxed{\frac{-36}{85}}$$

c. $\tan(x + y)$ $\rightarrow \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$$\tan x = \frac{15}{-8}$$
$$\tan y = \frac{-3}{4}$$

$$\frac{-\frac{15}{8} - \frac{3}{4}}{1 - \left(\frac{-15}{8}\right)\left(\frac{-3}{4}\right)} = \boxed{\frac{84}{13}}$$

d. $\cos(y - x)$ $\rightarrow \cos y \cos x + \sin y \sin x$

$$\left(\frac{4}{5}\right)\left(\frac{-8}{17}\right) + \left(\frac{-3}{5}\right)\left(\frac{15}{17}\right)$$

$$-\frac{32}{85} - \frac{45}{85} = \boxed{\frac{-77}{85}}$$

e. What quadrant is $(x + y)$ **in?** __Q1__

f. What quadrant is $(y - x)$ **in?** __Q3__

g. $\sin 2x$

$* 2\sin x \cos x$

$$\rightarrow 2\left(\frac{15}{17}\right)\left(\frac{-8}{17}\right) = \boxed{\frac{-240}{289}}$$

h. $\cos 2x$

$* 2\cos^2 x - 1$

$$\rightarrow 2\left(\frac{-8}{17}\right)^2 - 1$$

$$= \boxed{\frac{-161}{289}}$$

i. $\tan 2x$ $* \dfrac{2\tan x}{1 - \tan^2 x}$

$$\frac{2\left(\frac{-15}{8}\right)}{1 - \left(\frac{-15}{8}\right)^2} \rightarrow \boxed{\frac{240}{161}}$$

j. What quadrant is $2x$ **in?** __Q3__