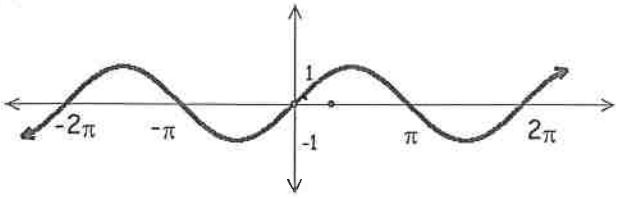
4.16 Trig Inverses and Principal Values

Recall that for a graph to be considered a function, it must pass the vertical line test. For a function to have an inverse, it also must pass the horizontal line test (meaning if you switch the x's and y's, the inverse would pass the vertical line test)

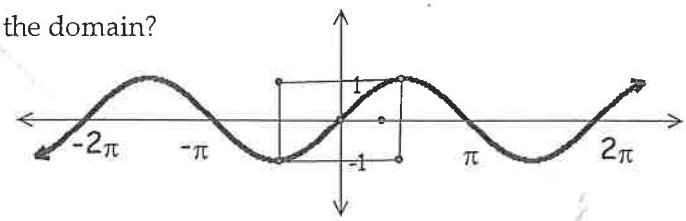
Recall the properties of an inverse function: 1) x and y switch, so the domain and range switch

2) the graph is reflected over y=x.

Does the graph of sin(x) pass the vertical line test? The horizontal line test?



What if we restrict the domain?

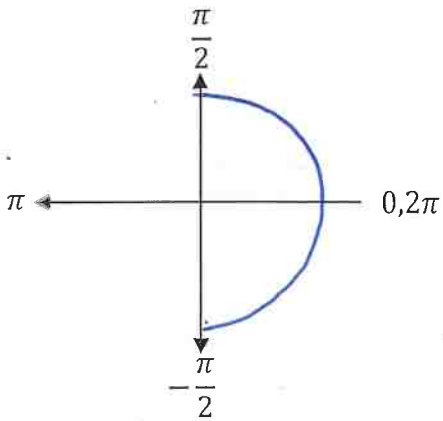


If we restrict the domain of sin(x) from $-\frac{\pi}{2}$ *to* $\frac{\pi}{2}$, the restricted portion would pass the horizontal line test, so the inverse is a function. We refer to these restricted values of x as the Principal Values of sinx.

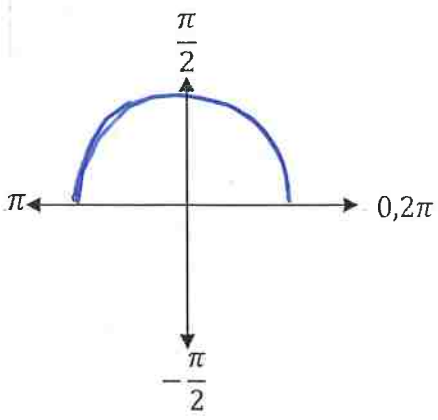
Notations for inverses with principal values: $Sin^{-1}x$ *or* $Arcsinx$, $Cos^{-1}x$ *or* $Arccosx$, and $Tan^{-1}x$ *or* $Arctanx$

Notice the capital letters at the beginning; they indicate you're finding principal values

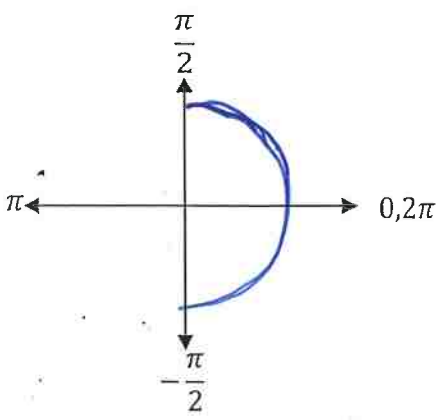
Here's how the domain restrictions look with respect to the unit circle:



$Sin^{-1}x$ *or* $Arcsinx$

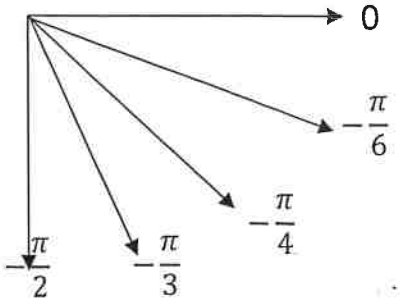


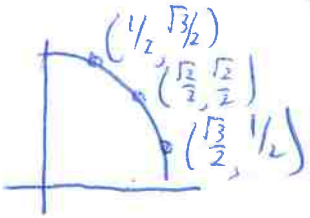
$Cos^{-1}x$ *or* $Arccosx$



$Tan^{-1}x$ *or* $Arctanx$

Pay special attention to QIV as these are not the values you see on the unit circle

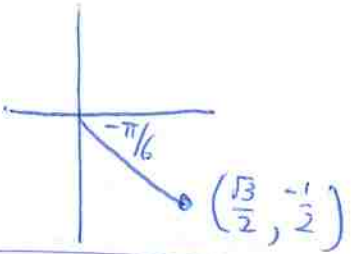




Example 1: Find $Arcsin\left(-\frac{1}{2}\right)$

a) Where do we look for *Arcsin*?

b) What angle has $sin = -\frac{1}{2}$?

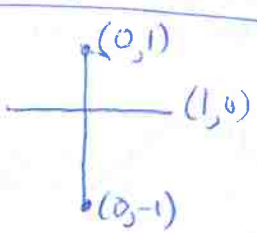


$\theta = -\pi/6$

Example 2: Find $Tan^{-1}(0)$

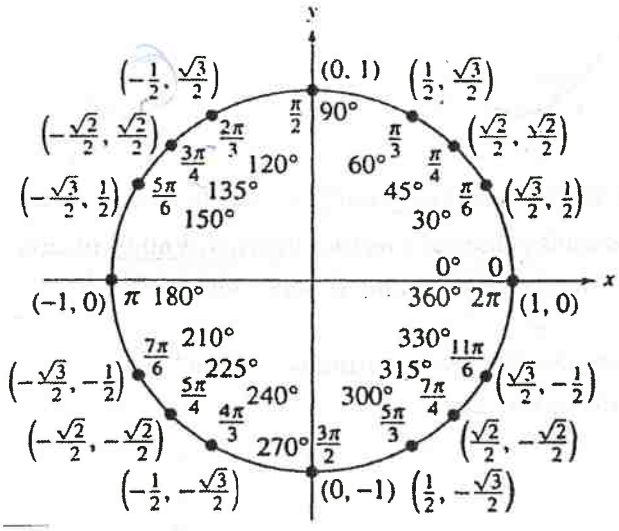
a) Where do we look for Tan^{-1}?

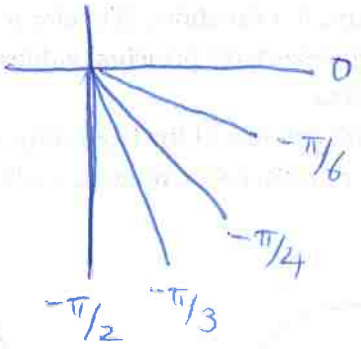
b) What angle has $tan = 0$?



$\theta = 0$

Refer to the unit circle to find Principal Values in degrees and radians.





a. $Arcsin(-1)$

This problem asks us to find the angle that has a sine value of -1

$Sin\theta = -1$

$\theta = -\pi/2$

b. $Arccos\left(-\frac{\sqrt{2}}{2}\right)$

This problem asks us to find the angle that has a cosine value of $-\frac{\sqrt{2}}{2}$

$Cos\theta = -\frac{\sqrt{2}}{2}$

$\theta = 3\pi/4$

c. $Arctan(1)$

This problem asks us to find the angle that has a tangent value of 1

$Tan\theta = 1$

$\theta = \pi/4$

d. $Sin^{-1}(0)$

$\theta = 0$

e. $Cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \pi/6$

f. $Tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\theta = -\pi/6$

g. $Arcsin\left(\frac{\sqrt{2}}{2}\right)$

$\theta = \pi/4$

h. $Arccos\left(-\frac{1}{2}\right)$

$\theta = \frac{2\pi}{3}$

i. $Arctan(-\sqrt{3})$ $-\frac{\sqrt{3}}{1}$

$\theta = -\pi/3$

4.16 Practice- Trig Inverses & Principal Values **Name:** _____

Find the exact principal value for each trig inverse function. **Date:** _____

1. $Arcsin\left(-\frac{\sqrt{3}}{2}\right)$ $\theta = -\pi/3$

2. $Cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\theta = \frac{3\pi}{4}$

3. $Arctan(1)$ $\theta = \frac{\pi}{4}$

4. $Arccos\left(\frac{\sqrt{3}}{2}\right)$ $\theta = \frac{\pi}{6}$

5. $Sin^{-1}(-1)$ $-\pi/2$

6. $Arctan(\sqrt{3})$ $\theta = \pi/3$

7. $Arctan(0)$ $\theta = 0$

8. $Sin^{-1}(3)$ undefined

9. $Arccos(-1)$ $\theta = \pi$

10) $\theta = -\pi/6$

10. $Arctan\left(-\frac{\sqrt{3}}{3}\right)$ or $-\frac{1}{\sqrt{3}}$

$\rightarrow -\frac{1}{\sqrt{3}}$

$\arctan\left(\frac{-1}{\sqrt{3}}\right)$

$\theta = -\pi/6$

11. $Arcsin(1)$ $\theta = \pi/2$

12. $Cos^{-1}\left(-\frac{1}{2}\right)$ $\theta = \frac{2\pi}{3}$