4.1, 4.2, 4.6 Help Session @7:30am 1/12/2023 (Thurs)

1) $\int 2x^3 - 2(3-x)^2 + \frac{5}{\sqrt{x^7}} - e \, dx$

2) $\int \frac{5x - 2\sqrt[3]{x} + x^5}{4\sqrt{x}} \, dx$

3) $\int 5\sec x\tan x - \cos x + \csc^2 x + 6\sin x \, dx$

4) $f(x)=4-x^2$ from $[-1, 1]$ Use 3 midpoint Rectangl

5) Given $f(x) = 3 - 2x^2$ $[-1, 1]$ Use limit definition of Area

6) Find $f(x)$ given $f''(x) = 3x^3 - 4x^2 + x - 7$

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1) $\int 2x^3 - 2(3-x)^2 + \frac{5}{\sqrt{x^7}} - e\,dx$

$(3-x)(3-x)$

$\int 2x^3 - 2(9-6x+x^2) + \frac{5}{x^{7/2}} - e\,dx$

$\int 2x^3 - 18 + 12x - 2x^2 + 5x^{-7/2 + \frac{2}{2}} - e\,dx$

$\frac{2x^4}{4} - 18x + \frac{12x^2}{2} - \frac{2x^3}{3} + \frac{5x^{-5/2}}{-5/2} - ex + C$

$-\frac{2}{5}(5)x^{-5/2}$

$$\boxed{\frac{1}{2}x^4 - 18x + 6x^2 - \frac{2}{3}x^3 - \frac{2}{x^{5/2}} - ex + C}$$

2) $\int \frac{5x - 2\sqrt[3]{x} + x^5}{4\sqrt{x}}\,dx$

$\int \frac{5x - 2x^{1/3} + x^5}{4x^{1/2}}\,dx$

$\int \frac{5x}{4x^{1/2}} - \frac{2x^{1/3}}{4x^{1/2}} + \frac{x^5}{4x^{1/2}}\,dx$

$\frac{2}{6} - \frac{3}{6}$ $\quad \frac{10}{2} - \frac{1}{2}$

$\int \frac{5}{4}x \cdot x^{-1/2} - \frac{1}{2}x^{1/3} \cdot x^{-1/2} + \frac{1}{4}x^5 \cdot x^{-1/2}\,dx$

$\int \frac{5}{4}x^{1/2} - \frac{1}{2}x^{-1/6} + \frac{1}{4}x^{9/2}\,dx$

$\frac{\frac{5}{4}x^{3/2}}{3/2} - \frac{\frac{1}{2}x^{5/6}}{5/6} + \frac{\frac{1}{4}x^{11/2}}{11/2} + C$

$\frac{2}{3} \cdot \frac{5}{4}x^{3/2} - \frac{6}{5} \cdot \frac{1}{2}x^{5/6} + \frac{2}{11} \cdot \frac{1}{4}x^{11/2} + C$

$$\boxed{\frac{5}{6}x^{3/2} - \frac{3}{5}x^{5/6} + \frac{1}{22}x^{11/2} + C}$$

3) $\int 5\sec x\tan x - \cos x + \csc^2 x + 6\sin x \, dx$

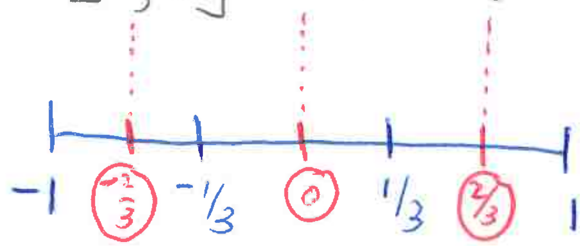
$$\boxed{5\sec x - \sin x - \cot x - 6\cos x + C}$$

4) $f(x) = 4 - x^2$ from $[-1, 1]$ Use 3 midpoint Rectangles

$W = \frac{b-a}{n} \rightarrow \frac{1-(-1)}{3} = \frac{2}{3}$

$\frac{-1 - \frac{1}{3}}{2} \rightarrow \frac{1}{2}\left(-\frac{3}{3} - \frac{1}{3}\right)$

$\rightarrow \frac{1}{2}\left(-\frac{4}{3}\right) \rightarrow -\frac{2}{3}$



$\text{Area} = \frac{2}{3}f(-2/3) + \frac{2}{3}f(0) + \frac{2}{3}f(2/3)$

$\text{Area} = \frac{2}{3}\left[3.556 + 4 + 3.556\right]$

$$\boxed{\text{Area} = 7.407}$$

5) Given $f(x) = 3 - 2x^2$ $[-1, 1]$ Use limit definition of Area

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{b-a}{n} \cdot f\left(a + \frac{b-a}{n} i\right) \right]$$

$$w = \frac{b-a}{n} \to \frac{1-(-1)}{n} = \frac{2}{n}$$

$$\sum_{i=1}^{n} \left[\frac{2}{n} \cdot f\left(-1 + \frac{2}{n} i\right) \right]$$

$$f(x) = 3 - 2x^2$$

$$f(\quad) = 3 - 2(\quad)^2$$

$$f\left(-1 + \frac{2}{n} i\right) = 3 - 2\left(-1 + \frac{2}{n} i\right)^2$$

$$\sum_{i=1}^{n} \left[\frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n} i\right)^2\right] \right]$$

$$\frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n} i\right)\left(-1 + \frac{2}{n} i\right) \right]$$

$$\frac{2}{n} \left[3 - 2\left(1 - \frac{4}{n} i + \frac{4}{n^2} i^2\right) \right]$$

$$\frac{2}{n} \left[\cancel{3 - 2}^{\,1} + \frac{8}{n} i - \frac{8}{n^2} i^2 \right]$$

$$\sum_{i=1}^{n} \frac{2}{n} + \frac{16}{n^2} i - \frac{16}{n^3} i^2$$

$$\sum_{i=1}^{n} \frac{2}{n} + \sum_{i=1}^{n} \frac{16}{n^2} i - \sum_{i=1}^{n} \frac{16}{n^3} i^2$$

$$\frac{2}{n} \boxed{\sum_{i=1}^{n} 1} + \frac{16}{n^2} \boxed{\sum_{i=1}^{n} i} - \frac{16}{n^3} \boxed{\sum_{i=1}^{n} i^2}$$

$$\lim_{n\to\infty} \frac{2}{n} \cdot n + \frac{16}{n^2} \cdot \frac{n(n+1)}{2} - \frac{16}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n\to\infty} \frac{2n}{n} + \frac{16n^2 + 16n}{2n^2} - \frac{32n^3 + \ldots}{6n^3} \to \boxed{\text{Area} = \frac{14}{3}}$$

$$\frac{2}{1} + \frac{16}{2} - \frac{32}{6} \longrightarrow \frac{14}{3}$$

6) Find $f(x)$ given $f''(x) = 3x^3 - 4x^2 + x - 7$

$$f''(x) = 3x^3 - 4x^2 + x - 7$$

$$f'(x) = \int 3x^3 - 4x^2 + x - 7\,dx$$

$$f'(x) = \frac{3x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} - 7x + C$$

$$f(x) = \int \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 - 7x + C\,dx$$

(or $+k$)

$$f(x) = \frac{3}{4}\cdot\frac{x^5}{5} - \frac{4}{3}\cdot\frac{x^4}{4} + \frac{1}{2}\cdot\frac{x^3}{3} - \frac{7x^2}{2} + Cx + C_2$$

$$\boxed{f(x) = \frac{3}{20}x^5 - \frac{1}{3}x^4 + \frac{1}{6}x^3 - \frac{7}{2}x^2 + Cx + C_2}$$

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$f(x) = \int \frac{3}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 - 7x + C\,dx$

(or $+K$)

$f(x) = \frac{3}{4} \cdot \frac{x^5}{5} - \frac{4}{3} \cdot \frac{x^4}{4} + \frac{1}{2} \cdot \frac{x^3}{3} - \frac{7x^2}{2} + Cx + C_2$

$$\boxed{f(x) = \frac{3}{20}x^5 - \frac{1}{3}x^4 + \frac{1}{6}x^3 - \frac{7}{2}x^2 + Cx + C_2}$$

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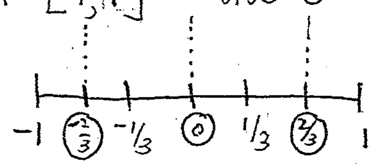

$\frac{-1-\frac{1}{3}}{2} \to \frac{1}{2}\left(\frac{-3}{3} - \frac{1}{3}\right)$

$\to \frac{1}{2}\left(-\frac{4}{3}\right) \to \frac{-2}{3}$

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$\sum_{i=1}^{n} \left[\frac{2}{n} \cdot f\left(-1 + \frac{2}{n}i\right)\right]$

$f(x) = 3 - 2x^2$

$f(\ \ ) = 3 - 2(\ \ )^2$

$f\left(-1 + \frac{2}{n}i\right) = 3 - 2\left(-1 + \frac{2}{n}i\right)^2$

$\sum_{i=1}^{n} \left[\frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n}i\right)^2\right]\right]$

$\frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n}i\right)\left(-1 + \frac{2}{n}i\right)\right]$

$\frac{2}{n}\left[3 - 2\left(1 - \frac{4}{n}i + \frac{4}{n^2}i^2\right)\right]$

$\frac{2}{n}\left[\cancel{3}^{1} \cancel{-2} + \frac{8}{n}i - \frac{8}{n^2}i^2\right]$

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$\frac{2}{1} + \frac{16}{2} - \frac{32}{6} \to \frac{14}{3}$

$$\boxed{Area = \frac{14}{3}}$$