Key

AP Calculus AB 4-1, 4-2, 4-6 Morning Review WS #3

Calculators permitted.

1. Find the sum: $\sum_{i=1}^{3}\left[(2i+1)^2-(3i+1)^3\right] = (2(1)+1)^2-(3(1)+1)^3+(2(2)+1)^2-(3(2)+1)^3+(2(3)+1)^2-(3(3)+1)^3$

$= 3^2-4^3+5^2-7^3+7^2-10^3 = \boxed{-1324}$

2. Use Sigma notation to write the sum: $\frac{7\sqrt{3}}{27}+\frac{7\sqrt{4}}{64}+\frac{7\sqrt{5}}{125}+\frac{7\sqrt{6}}{216}$.

3. Use 4 middle rectangles to approximate the area of the region bounded by $f(x) = 3 + 2x^2$, the x-axis, $x = 1$, and $x = 7$.

$w = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2}$

$1 \quad 7/4 \quad 5/2 \quad 13/4 \quad 8/2 \quad 19/4 \quad 11/2 \quad 25/4 \quad 14/2$

$\text{Area} \approx \frac{3}{2}\cdot f(7/4)+\frac{3}{2}\cdot f(13/4)+\frac{3}{2}\cdot f\left(\frac{19}{4}\right)+\frac{3}{2}f\left(\frac{25}{4}\right)$

$= \frac{3}{2}(9.125)+\frac{3}{2}(24.125)+\frac{3}{2}(48.125)+\frac{3}{2}(81.125) = \boxed{243.75}$

4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [1, 15]

5 5 4

1	5	6	8	11	13	15
4	2	7	3	1	6	5

$5(2)+5(3)+4(6) = \boxed{49}$

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [5, 11]

1 2 3

5	6	8	11
2	7	3	1

$1(7)+2(3)+3(1) = \boxed{16}$

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 15]

$*\text{Area} = \frac{w}{2}[h_1+h_2]$

2 3 2 2

6	8	11	13	15
7	3	1	6	5

$\frac{2}{2}[7+3]+\frac{3}{2}[3+1]+\frac{2}{2}[1+6]+\frac{2}{2}[6+5]$

$10+\frac{3}{2}(4)+1(7)+1(11) = \boxed{34}$

5. Given the region bounded by $g(x) = 3 - 2x^2$, the x-axis, $x = -1$, and $x = 1$. Use the **limit definition** to find the exact area of the region.

$w = \frac{1-(-1)}{n} = \frac{2}{n}$

$\text{Area} = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\cdot f\left[-1+\frac{2}{n}i\right]$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\cdot\left[3-2\left(-1+\frac{2}{n}i\right)^2\right]$

$\sum_{i=1}^{n}\frac{2}{n}\left[3-2\left(1-\frac{4}{n}i+\frac{4}{n^2}i^2\right)\right]$

$\frac{2}{n}\left[3-2+\frac{8}{n}i-\frac{8}{n^2}i^2\right]$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left[1+\frac{8}{n}i-\frac{8}{n^2}i^2\right]$

$\sum_{i=1}^{n}\frac{2}{n}+\frac{16}{n^2}i-\frac{16}{n^3}i^2$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}+\sum\frac{16}{n^2}i-\sum\frac{16}{n^3}i^2$

$\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}1+\frac{16}{n^2}\sum_{i=1}^{n}i-\frac{16}{n^3}\sum_{i=1}^{n}i^2$

$\lim_{n\to\infty}\frac{2}{n}(n)+\frac{16}{n^2}\cdot\frac{n(n+1)}{2}-\frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty}\frac{2n}{n}+\frac{16n^2}{2n^2}-\frac{32n^3}{6n^3}$

$2+8-\frac{32}{6} = \boxed{\frac{14}{3}}$

Find the general antiderivative of $g(x)$. (Find $\int g(x)dx$)

6. $g(x) = x(2x-1)^2$

$\int x(2x-1)^2 dx$

$\int x(2x-1)(2x-1)dx$

$\int x(4x^2-4x+1)dx$

$\int 4x^3 - 4x^2 + x\, dx$

$\frac{4x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + C$

$\boxed{x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} + C}$

7. $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

$\int 4x^{-1/3} - x^{1/2} + 3x^2 - \frac{1}{3}x^{-4} dx$

$4\left(\frac{x^{2/3}}{2/3}\right) - \frac{x^{3/2}}{3/2} + \frac{3x^3}{3} - \frac{1}{3} \cdot \frac{x^{-3}}{-3} + C$

$\boxed{6x^{2/3} - \frac{2}{3}x^{3/2} + x^3 + \frac{1}{9x^3} + C}$

8. $g(x) = \frac{x^3 - 2\sqrt{x} + \sqrt[4]{x}}{\sqrt{x}}$

$\int (x^3 - 2x^{1/2} + x^{1/4})x^{-1/2} dx$

$\int x^{5/2} - 2 + x^{-1/4} dx$

$\frac{x^{7/2}}{7/2} - 2x + \frac{x^{3/4}}{3/4} + C$

$\boxed{\frac{2}{7}x^{7/2} - 2x + \frac{4}{3}x^{3/4} + C}$

9. Find the **general** expression of $f(x)$ if $f''(x) = 2x^3 + 3x^2 + x - 1$

$f'(x) = \int 2x^3 + 3x^2 + x - 1\, dx$

$f'(x) = \frac{2x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2} - x + c$

$f(x) = \frac{1}{2} \cdot \frac{x^5}{5} + \frac{x^4}{4} + \frac{1}{2} \cdot \frac{x^3}{3} - \frac{x^2}{2} + Cx + k$

$\boxed{f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{1}{2}x^2 + Cx + k}$

10. Find the **specific** expression of $f(x)$ if $f''(x) = 12x^2 + 18x - 4$, $f'(-1) = 9$, and $f(1) = 3$

$f'(x) = \int 12x^2 + 18x - 4\, dx$

$f'(x) = \frac{12x^3}{3} + \frac{18x^2}{2} - 4x + C$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$

$9 = -4 + 9 + 4 + C$

$0 = C$

$f'(x) = 4x^3 + 9x^2 - 4x$

$f(x) = \int 4x^3 + 9x^2 - 4x\, dx$

$f(x) = \frac{4x^4}{4} + \frac{9x^3}{3} - \frac{4x^2}{2} + k$

$f(x) = x^4 + 3x^3 - 2x^2 + k$

$3 = (1)^4 + 3(1)^3 - 2(1)^2 + k$

$3 = 1 + 3 - 2 + k$

$1 = k$

$\boxed{f(x) = x^4 + 3x^3 - 2x^2 + 1}$

4.1, 4.2, 4.6 Formula Sheet:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} \boldsymbol{ca_i} = c\sum_{i=1}^{n} \boldsymbol{a_i}$

Area using Limit Definition

$$\lim_{n\to\infty}\sum_{i=1}^{n}(width) * f(left\ endpoint + width * i)$$

$$\text{width} = \frac{b-a}{n}$$

Trapezoid Area:

$$Area = \frac{1}{2}w(h_1 + h_2)$$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int \sin u\, du = -\cos u + C \qquad \int \cos u\, du = \sin u + C$$

$$\int \sec^2 u\, du = \tan u + C \qquad \int \sec u \tan u\, du = \sec u + C$$

$$\int \csc^2 u\, du = -\cot u + C \qquad \int \csc u \cot u\, du = -\csc u + C$$