

Key

AP Calculus AB 4-1, 4-2, 4-6 Morning Review WS #3

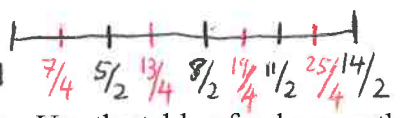
Calculators permitted.

1. Find the sum:  $\sum_{i=1}^3 [(2i+1)^2 - (3i+1)^3] = (2(1)+1)^2 - (3(1)+1)^3 + (2(2)+1)^2 - (3(2)+1)^3 + (2(3)+1)^2 - (3(3)+1)^3$   
 $= 3^2 - 4^3 + 5^2 - 7^3 + 7^2 - 10^3 = -1324$

2. Use Sigma notation to write the sum:  $\frac{7\sqrt{3}}{27} + \frac{7\sqrt{4}}{64} + \frac{7\sqrt{5}}{125} + \frac{7\sqrt{6}}{216}$

3. Use 4 middle rectangles to approximate the area of the region bounded by  $f(x) = 3 + 2x^2$ , the x-axis,  $x = 1$ , and  $x = 7$ .

$w = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2}$



Area  $\approx \frac{3}{2} \cdot f(7/4) + \frac{3}{2} \cdot f(5/2) + \frac{3}{2} \cdot f(13/4) + \frac{3}{2} \cdot f(8/2)$   
 $= \frac{3}{2}(9.125) + \frac{3}{2}(24.125) + \frac{3}{2}(48.125) + \frac{3}{2}(81.125) = 243.75$

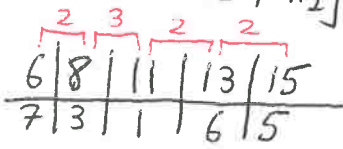
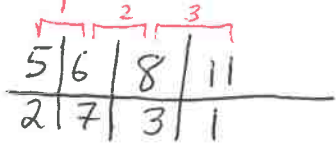
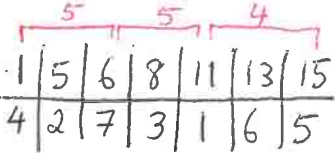
4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [1, 15]

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [5, 11]

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 15]



$5(2) + 5(3) + 4(6) = 49$

$1(7) + 2(3) + 3(1) = 16$

$\frac{2}{2}[7+3] + \frac{3}{2}[3+1] + \frac{2}{2}[1+6] + \frac{2}{2}[6+5]$   
 $10 + \frac{3}{2}(4) + 1(7) + 1(11) = 34$

5. Given the region bounded by  $g(x) = 3 - 2x^2$ , the x-axis,  $x = -1$ , and  $x = 1$ . Use the **limit definition** to find the exact area of the region.

$w = \frac{1-(-1)}{n} = \frac{2}{n}$

Area =  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \cdot f\left[-1 + \frac{2}{n}i\right]$

$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \cdot \left[3 - 2\left(-1 + \frac{2}{n}i\right)^2\right]$

$\sum_{i=1}^n \frac{2}{n} \left[3 - 2\left(1 - \frac{4}{n}i + \frac{4}{n^2}i^2\right)\right]$

$\frac{2}{n} \left[3 - 2 + \frac{8}{n}i - \frac{8}{n^2}i^2\right]$

$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left[1 + \frac{8}{n}i - \frac{8}{n^2}i^2\right]$

$\sum_{i=1}^n \frac{2}{n} + \frac{16}{n^2}i - \frac{16}{n^3}i^2$

$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} + \sum \frac{16}{n^2}i - \sum \frac{16}{n^3}i^2$

$\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n 1 + \frac{16}{n^2} \sum_{i=1}^n i - \frac{16}{n^3} \sum_{i=1}^n i^2$

$\lim_{n \rightarrow \infty} \frac{2}{n}(n) + \frac{16}{n^2} \cdot \frac{n(n+1)}{2} - \frac{16}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$

$\lim_{n \rightarrow \infty} \frac{2n}{n} + \frac{16n^2}{2n^2} - \frac{32n^3}{6n^3}$

$2 + 8 - \frac{32}{6} = \frac{14}{3}$

Find the general antiderivative of  $g(x)$ . (Find  $\int g(x)dx$ )

6.  $g(x) = x(2x - 1)^2$

$$\int x(2x-1)^2 dx$$

$$\int x(2x-1)(2x-1) dx$$

$$\int x(4x^2 - 4x + 1) dx$$

$$\int 4x^3 - 4x^2 + x dx$$

$$\frac{4x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + C$$

$$x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} + C$$

7.  $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

$$\int 4x^{-1/3} - x^{1/2} + 3x^2 - \frac{1}{3}x^{-4} dx$$

$$4 \left( \frac{x^{2/3}}{2/3} \right) - \frac{x^{3/2}}{3/2} + \frac{3x^3}{3} - \frac{1}{3} \frac{x^{-3}}{-3} + C$$

$$6x^{2/3} - \frac{2}{3}x^{3/2} + x^3 + \frac{1}{9}x^{-3} + C$$

8.  $g(x) = \frac{x^3 - 2\sqrt{x} + \sqrt[4]{x}}{\sqrt{x}}$

$$\int (x^3 - 2x^{1/2} + x^{1/4}) x^{-1/2} dx$$

$$\int x^{5/2} - 2 + x^{-1/4} dx$$

$$\frac{x^{7/2}}{7/2} - 2x + \frac{x^{3/4}}{3/4} + C$$

$$\frac{2}{7}x^{7/2} - 2x + \frac{4}{3}x^{3/4} + C$$

9. Find the **general** expression of  $f(x)$  if  $f''(x) = 2x^3 + 3x^2 + x - 1$

$$f'(x) = \int 2x^3 + 3x^2 + x - 1 dx$$

$$f(x) = \frac{1}{2} \cdot \frac{x^5}{5} + \frac{x^4}{4} + \frac{1}{2} \cdot \frac{x^3}{3} - \frac{x^2}{2} + Cx + k$$

$$f'(x) = \frac{2x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2} - x + C$$

$$f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{1}{2}x^2 + Cx + k$$

10. Find the **specific** expression of  $f(x)$  if  $f''(x) = 12x^2 + 18x - 4$ ,  $f'(-1) = 9$ , and  $f(1) = 3$

$$f'(x) = \int 12x^2 + 18x - 4 dx$$

$$f'(x) = 4x^3 + 9x^2 - 4x$$

$$f(x) = x^4 + 3x^3 - 2x^2 + k$$

$$f'(x) = \frac{12x^3}{3} + \frac{18x^2}{2} - 4x + C$$

$$f(x) = \int 4x^3 + 9x^2 - 4x dx$$

$$3 = (1)^4 + 3(1)^3 - 2(1)^2 + k$$

$$3 = 1 + 3 - 2 + k$$

$$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$$

$$f(x) = \frac{4x^4}{4} + \frac{9x^3}{3} - \frac{4x^2}{2} + k$$

$$\underline{\underline{1 = k}}$$

$$9 = -4 + 9 + 4 + C$$

$$0 = C$$

$$f(x) = x^4 + 3x^3 - 2x^2 + 1$$

#### 4.1, 4.2, 4.6 Formula Sheet:

##### Summation Formulas:

$$1) \sum_{i=1}^n 1 = n$$

$$2) \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$3) \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$4) \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

$$5) \sum_{i=1}^n c a_i = c \sum_{i=1}^n a_i$$

##### Area using Limit Definition

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (\text{width}) * f(\text{left endpoint} + \text{width} * i)$$

$$\text{width} = \frac{b-a}{n}$$

##### Trapezoid Area:

$$\text{Area} = \frac{1}{2} w (h_1 + h_2)$$

##### Integral Formulas:

##### Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

##### Trig Integrals:

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \csc u \cot u du = -\csc u + C$$