

Key

AP Calculus AB 4-1, 4-2, 4-6 Quiz Review #1  
Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^4 [(i+1)^2 - (2-i)^3] = (2+1)^2 - (2-2)^3 + (3+1)^2 - (2-3)^3 + (4+1)^2 - (2-4)^3$$

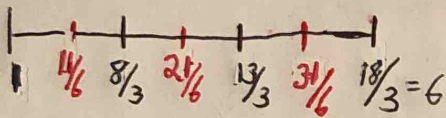
$$= 9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$$

2. Use Sigma notation to write the sum:  $\frac{2}{\sqrt[3]{5-2}} + \frac{4}{\sqrt[3]{5-4}} + \frac{6}{\sqrt[3]{5-6}} + \frac{8}{\sqrt[3]{5-8}}$

$$\sum_{i=1}^4 \frac{2i}{\sqrt[3]{5-2i}}$$

3. Use 3 middle rectangles to approximate the area of the region bounded by  $f(x) = x^2 + 3$ , the x-axis,  $x = 1$ , and  $x = 6$ .

width =  $\frac{b-a}{n} \rightarrow \frac{6-1}{3} = \frac{5}{3}$



$$\text{Area} = \frac{5}{3} \cdot f(1\frac{1}{6}) + \frac{5}{3} f(2\frac{2}{6}) + \frac{5}{3} (f(3\frac{1}{6}))$$

$$= \frac{5}{3} \cdot \left[ \left(\frac{11}{6}\right)^2 + 3 \right] + \frac{5}{3} \left[ \left(\frac{21}{6}\right)^2 + 3 \right] + \frac{5}{3} \left[ \left(\frac{31}{6}\right)^2 + 3 \right]$$

$$= \frac{5}{3} (6.361) + \frac{5}{3} (15.25) + \frac{5}{3} (29.69) = \boxed{85.509}$$

4. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
f(x)	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on  $[0, 7]$

x	0	4	6	7
f(x)	5	3	2	3

$$\text{Area} = 4 \cdot f(0) + 2 \cdot f(4) + 1 \cdot f(6)$$

$$\text{Area} = 4(5) + 2(3) + 1(2)$$

$$= 20 + 6 + 2 = \boxed{28}$$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on  $[0, 10]$

x	0	4	6	7	10
f(x)	5	3	2	3	5

$$\text{Area} = 6 \cdot f(4) + 4 \cdot f(7)$$

$$= 6(3) + 4(3)$$

$$= \boxed{30}$$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on  $[4, 10]$

x	4	6	7	10
f(x)	3	2	3	5

$$\text{Area} = 2 \cdot f(6) + 1 \cdot f(7) + 3 \cdot f(10)$$

$$= 2(2) + 1(3) + 3(5) = \boxed{22}$$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on  $[0, 7]$

x	0	4	6	7
f(x)	5	3	2	3

$$* \text{Area} = \frac{1}{2} \cdot w \cdot [h_1 + h_2]$$

$$\text{Area} = \frac{1}{2} \cdot 4 [f(0) + f(4)] + \frac{1}{2} (2) [f(4) + f(6)] + \frac{1}{2} (1) [f(6) + f(7)]$$

$$A = 2(5+3) + 1(3+2) + \frac{1}{2}(2+3)$$

$$A = \boxed{23.5}$$

5. Given the region bounded by  $g(x) = 6 - x^2$ , the  $x$ -axis,  $x = -1$ , and  $x = 2$ . Use the limit definition to find the exact area of the region.

$$W = \frac{2 - (-1)}{n} = \frac{3}{n}$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{width} \cdot f(a + \text{width} \cdot i) = \frac{15}{n} \sum_{i=1}^n 1 + \frac{18}{n^2} \sum_{i=1}^n i - \frac{27}{n^3} \sum_{i=1}^n i^2$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \cdot f\left(-1 + \frac{3}{n}i\right)$$

$$\frac{3}{n} \left[ 6 - \left( -1 - \frac{6i}{n} + \frac{9i^2}{n^2} \right) \right]$$

$$\lim_{n \rightarrow \infty} \frac{15}{n}(n) + \frac{18}{n^2} \cdot \frac{n(n+1)}{2} - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[ 6 - \left( -1 + \frac{3}{n}i \right)^2 \right]$$

$$\frac{3}{n} \left[ 5 + \frac{6i}{n} - \frac{9i^2}{n^2} \right]$$

$$\lim_{n \rightarrow \infty} \frac{15n}{n} + \frac{18n^2}{2n^2} - \frac{54n^3 + \dots}{6n^3}$$

$$A = \frac{3}{n} \left[ 6 - \left( -1 + \frac{3}{n}i \right) \left( -1 + \frac{3}{n}i \right) \right]$$

$$\lim_{n \rightarrow \infty} \sum \frac{15}{n} + \frac{18i}{n^2} - \frac{27i^2}{n^3}$$

$$15 + 9 - 9 = \boxed{15}$$

Find the most general antiderivative of  $h(x)$ . (Find  $\int h(x) dx$ )

6.  $h(x) = 5x^4 - \pi + \frac{1}{2\sqrt{x}} + \frac{1}{3x^3}$

$$\int 5x^4 - \pi + \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-3} dx$$

$$\frac{5x^5}{5} - \pi x + \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + \frac{1}{3} \cdot \frac{x^{-2}}{-2} + C$$

$$\boxed{x^5 - \pi x + x^{1/2} - \frac{1}{6x^2} + C}$$

7.  $h(x) = -2\cos x + 5\sin x - 5\csc x \cot x$

$$\int -2\cos x + 5\sin x - 5\csc x \cot x dx$$

$$\boxed{-2\sin x - 5\cos x + 5\csc x + C}$$

8. Find the most general expression of  $f(x)$  if  $f'''(x) = 4x^3 - 5x^2 + 3x - 6$ .

$$f'(x) = \int 4x^3 - 5x^2 + 3x - 6 dx$$

$$f'(x) = \frac{4x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + C$$

$$f'(x) = x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x + C$$

$$f(x) = \int x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x + C dx$$

$$f(x) = \frac{x^5}{5} - \frac{5}{3} \cdot \frac{x^4}{4} + \frac{3}{2} \cdot \frac{x^3}{3} - \frac{6x^2}{2} + Cx + k$$

$$\boxed{f(x) = \frac{1}{5}x^5 - \frac{5x^4}{12} + \frac{1}{2}x^3 - 3x^2 + Cx + k}$$

9. Find the specific expression of  $f(x)$  if  $f(x) = \int g(x) dx$ ,  $g(x) = 3x^2 - 4x$ , and  $f(-1) = 2$

$$\int 3x^2 - 4x dx$$

$$f(x) = \frac{3x^3}{3} - \frac{4x^2}{2} + C$$

$$2 = (-1)^3 - 2(-1)^2 + C$$

plug in (-1, 2)  
to solve for C

$$2 = -1 - 2 + C$$

$$2 = -3 + C$$

$$\underline{\underline{5 = C}}$$

$$\boxed{f(x) = x^3 - 2x^2 + 5}$$