Key

## AP Calculus AB 4-1,4-2, 4-6 Quiz Review #1

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}[(i+1)^2-(2-i)^3] = (2+1)^2-(2-2)^3+(3+1)^2-(2-3)^3+(4+1)^2-(2-4)^3$$
$$= 9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$$

2. Use Sigma notation to write the sum: $\frac{2}{\sqrt[3]{5-2}}+\frac{4}{\sqrt[3]{5-4}}+\frac{6}{\sqrt[3]{5-6}}+\frac{8}{\sqrt[3]{5-8}}$

$$\sum_{i=1}^{4}\frac{2i}{\sqrt[3]{5-2i}}$$

3. Use 3 middle rectangles to approximate the area of the region bounded by $f(x)=x^2+3$, the $x$-axis, $x=1$, and $x=6$.

$\text{width} = \frac{b-a}{n} \rightarrow \frac{6-1}{3} = \frac{5}{3}$

1 $\;1\frac{5}{6}\;$ $8/3$ $\;2\frac{1}{6}\;$ $13/3$ $\;3\frac{1}{6}\;$ $18/3=6$

$$\text{Area} = \frac{5}{3}\cdot f(11/6) + \frac{5}{3}f(21/6) + \frac{5}{3}(f(31/6))$$
$$= \frac{5}{3}\cdot\left[\left(\frac{11}{6}\right)^2+3\right] + \frac{5}{3}\left[\left(\frac{21}{6}\right)^2+3\right] + \frac{5}{3}\left[\left(\frac{31}{6}\right)^2+3\right]$$
$$= \frac{5}{3}(6.361) + \frac{5}{3}(15.25) + \frac{5}{3}(29.69) = \boxed{85.509}$$

4. Use the table of values on the right to estimate the below:

| $x$ | 0 | 4 | 6 | 7 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 5 | 3 | 2 | 3 | 5 |

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7]

| X | 0 | 4 | 6 | 7 |
|---|---|---|---|---|
| f(x) | 5 | 3 | 2 | 3 |

$\text{Area} = 4\cdot f(0) + 2f(4) + 1\cdot f(6)$

$\text{Area} = 4(5) + 2(3) + 1(2)$

$= 20 + 6 + 2 = \boxed{28}$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10]

$\overbrace{0 \quad 4 \quad 6}^{6}\;\overbrace{6 \quad 7 \quad 10}^{4}$

| X | 0 | 4 | 6 | 7 | 10 |
|---|---|---|---|---|---|
| f(x) | 5 | 3 | 2 | 3 | 5 |

$\text{Area} = 6\cdot f(4) + 4\cdot f(7)$

$= 6(3) + 4(3)$

$= \boxed{30}$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10]

2 1 3

| X | 4 | 6 | 7 | 10 |
|---|---|---|---|---|
| f(x) | 3 | 2 | 3 | 5 |

$\text{Area} = 2\cdot f(6) + 1\cdot f(7) + 3\cdot f(10)$

$= 2(2) + 1(3) + 3(5) = \boxed{22}$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7]

4 2 1

| X | 0 | 4 | 6 | 7 |
|---|---|---|---|---|
| f(x) | 5 | 3 | 2 | 3 |

$*\text{Area} = \frac{1}{2}\cdot w\cdot[h_1+h_2]$

$\text{Area} = \frac{1}{2}\cdot 4[f(0)+f(4)] + \frac{1}{2}(2)[f(4)+f(6)] + \frac{1}{2}(1)[f(6)+f(7)]$

$A = 2(5+3) + 1(3+2) + \frac{1}{2}(2+3)$

$\boxed{A = 23.5}$

5. Given the region bounded by g (x) = 6 – x², the x-axis, x = -1, and x = 2. Use the limit definition to find the exact area of the region.

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width}\cdot f(a+\text{width}\cdot i)$

$w = \frac{2-(-1)}{n} = \frac{3}{n}$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\cdot f(-1+\frac{3}{n}i)$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[6-(-1+\frac{3}{n}i)^2\right]$

$A = \frac{3}{n}\left[6-(-1+\frac{3}{n}i)(-1+\frac{3}{n}i)\right]$

$\frac{3}{n}\left[6-(1-\frac{6i}{n}+\frac{9i^2}{n^2})\right]$

$\frac{3}{n}\left[5+\frac{6i}{n}-\frac{9i^2}{n^2}\right]$

$\lim_{n\to\infty} \sum \frac{15}{n}+\frac{18i}{n^2}-\frac{27i^2}{n^3}$

$\lim_{n\to\infty} \sum \frac{15}{n}+\sum\frac{18i}{n^2}-\sum\frac{27i^2}{n^3}$

$= \frac{15}{n}\boxed{\sum_{i=1}^{n} 1} + \frac{18}{n^2}\boxed{\sum_{i=1}^{n} i} - \frac{27}{n^3}\boxed{\sum_{i=1}^{n} i^2}$

$\lim_{n\to\infty} \frac{15}{n}(n)+\frac{18}{n^2}\cdot\frac{n(n+1)}{2} - \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty} \frac{15n}{n}+\frac{18n^2}{2n^2}-\frac{54n^3+\dots}{6n^3}$

$15+9-9 = \boxed{15}$

Find the most general antiderivative of $h(x)$. (Find $\int h(x)dx$)

6. $h(x) = 5x^4 - \pi + \frac{1}{2\sqrt{x}} + \frac{1}{3x^3}$

$\int 5x^4 - \pi + \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-3}\,dx$

$\frac{5x^5}{5} - \pi x + \frac{1}{2}\cdot\frac{x^{1/2}}{1/2} + \frac{1}{3}\cdot\frac{x^{-2}}{-2} + C$

$\boxed{x^5 - \pi x + x^{1/2} - \frac{1}{6x^2} + C}$

7. h(x) = -2cosx + 5sinx – 5cscxcotx

$\int -2\cos x + 5\sin x - 5\csc x\cot x\,dx$

$\boxed{-2\sin x - 5\cos x + 5\csc x + C}$

8. Find the most general expression of $f(x)$ if $f''(x) = 4x^3 - 5x^2 + 3x - 6$.

$f'(x) = \int 4x^3 - 5x^2 + 3x - 6\,dx$

$f'(x) = \frac{4x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + C$

$f'(x) = x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x + C$

$f(x) = \int x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x + C\,dx$

$f(x) = \frac{x^5}{5} - \frac{5}{3}\cdot\frac{x^4}{4} + \frac{3}{2}\cdot\frac{x^3}{3} - \frac{6x^2}{2} + Cx + k$

$\boxed{f(x) = \frac{1}{5}x^5 - \frac{5x^4}{12} + \frac{1}{2}x^3 - 3x^2 + Cx + k}$

9. Find the specific expression of $f(x)$ if $f(x) = \int g(x)dx$, $g(x) = 3x^2 - 4x$, and $f(-1) = 2$

$\int 3x^2 - 4x\,dx$

$f(x) = \frac{3x^3}{3} - \frac{4x^2}{2} + C$

plug in (-1, 2) to solve for C

$2 = (-1)^3 - 2(-1)^2 + C$

$2 = -1 - 2 + C$

$2 = -3 + C$

$5 = C$

$\boxed{f(x) = x^3 - 2x^2 + 5}$