

Key

AP Calculus AB 4-1, 4-2, 4-6 Quiz Review WS #2

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^4 [(i+1)^2 + 3(2i-1)^3] = (2+1)^2 + 3(2(2)-1)^3 + (3+1)^2 + 3(2(3)-1)^3 + (4+1)^2 + 3(2(4)-1)^3$$

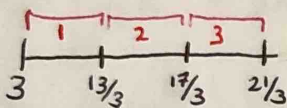
$$9 + 3(27) + 16 + 3(125) + 25 + 3(43)(3) = \boxed{1535}$$

2. Use Sigma notation to write the sum: $\frac{5-\sqrt{2}}{1} + \frac{5-\sqrt{4}}{4} + \frac{5-\sqrt{6}}{9} + \frac{5-\sqrt{8}}{16}$

$$\sum_{i=1}^4 \frac{5-\sqrt{2i}}{i^2}$$

3. Use 3 left rectangles to approximate the area of the region bounded by $f(x) = 1 + 2x^2$, the x-axis, $x=3$, and $x=7$.

$$w = \frac{b-a}{n} \rightarrow \frac{7-3}{3} = \frac{4}{3}$$



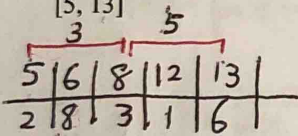
$$\text{Area} \approx \frac{4}{3} \cdot f(3) + \frac{4}{3} \cdot f(13/3) + \frac{4}{3} \cdot f(17/3)$$

$$\frac{4}{3}(19) + \frac{4}{3}(38.55) + \frac{4}{3}(65.22) \approx \boxed{163.696}$$

4. Use the table of values on the right to estimate the below:

x	2	5	6	8	12	13	14
f(x)	1	2	8	3	1	6	5

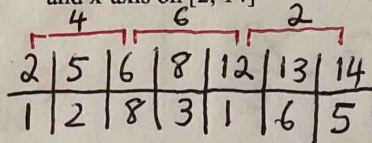
a. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on $[5, 13]$



$$3(8) + 5(1) = \boxed{29}$$

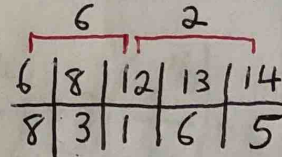
$$f(x) = 3 + 2(x)^2$$

b. Use 3 left-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on $[2, 14]$



$$4(1) + 6(8) + 2(1) = \boxed{54}$$

c. Use 2 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on $[6, 14]$



$$\frac{6}{2}[8+1] + \frac{2}{2}[1+5]$$

$$3(9) + 1(6) = \boxed{33}$$

$$\text{Area} = \frac{w}{2}[h_1 + h_2]$$

$$\frac{1}{2}w[h_1 + h_2]$$

5. Given the region bounded by $f(x) = 3 + 2x^2$, the x-axis, $x = -2$, and $x = 1$. Use the limit definition to find the exact area of the region.

$$\text{width} = \frac{1 - (-2)}{n} = \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \cdot f(-2 + \frac{3}{n}i)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \cdot \left[3 + 2\left(-2 + \frac{3}{n}i\right)^2 \right]$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[3 + 2\left(4 - \frac{12}{n}i + \frac{9}{n^2}i^2\right) \right]$$

\uparrow
 $\left(-2 + \frac{3}{n}i\right)\left(-2 + \frac{3}{n}i\right)$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left[3 + 8 - \frac{24}{n}i + \frac{18}{n^2}i^2 \right] \rightarrow \sum \frac{33}{n} - \frac{72}{n^2}i + \frac{54}{n^3}i^2$$

$$\sum \frac{33}{n} - \sum \frac{72}{n^2}i + \sum \frac{54}{n^3}i^2$$

$$\lim_{n \rightarrow \infty} \frac{33}{n} \left[\sum_{i=1}^n 1 \right] - \frac{72}{n^2} \left[\sum_{i=1}^n i \right] + \frac{54}{n^3} \left[\sum_{i=1}^n i^2 \right]$$

$$\lim_{n \rightarrow \infty} \frac{33}{n}(n) - \frac{72}{n^2} \left(\frac{n(n+1)}{2} \right) + \frac{54}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n \rightarrow \infty} \frac{33n}{n} - \frac{72n^2}{2n^2} + \frac{108n^3}{6n^3} = 33 - 36 + 18 = \boxed{15}$$

Find the general antiderivative of $g(x)$. (Find $\int g(x) dx$)

6. $g(x) = 3 \cos x - 5 \sin x + \csc x \cot x - 3\sqrt{x}$

$$\int 3 \cos x - 5 \sin x + \csc x \cot x - 3x^{1/2} dx$$

$$3(+\sin x) - 5(-\cos x) - \csc x - 3\left(\frac{x^{3/2}}{3/2}\right) + C$$

$$3 \sin x + 5 \cos x - \csc x - 2x^{3/2} + C$$

7. $g(x) = \frac{2}{3(\sqrt[5]{x})} - 3x^2 - \frac{1}{3e^4}$

$$G(x) = \int \frac{2}{3} x^{-1/5} - 3x^2 - \frac{1}{3e^4} dx$$

$$G(x) = \frac{2}{3} \left(\frac{x^{4/5}}{4/5}\right) - 3 \cdot \frac{x^3}{3} - \left(\frac{1}{3e^4}\right)x + C$$

$$\frac{2}{3} \cdot \frac{5}{4} x^{4/5} - x^3 - \frac{1}{3e^4} x + C$$

$$\frac{5}{6} x^{4/5} - x^3 - \frac{1}{3e^4} x + C$$

8. $g(x) = \frac{2x^3 - 5\sqrt{x} + 3(\sqrt[4]{x})}{x}$

$$g(x) = (2x^3 - 5x^{1/2} + 3x^{1/4})x^{-1}$$

$$g(x) = 2x^2 - 5x^{-1/2} + 3x^{-3/4}$$

$$\int 2x^2 - 5x^{-1/2} + 3x^{-3/4} dx$$

$$\frac{2x^3}{3} - 5\left(\frac{x^{1/2}}{1/2}\right) + 3\left(\frac{x^{1/4}}{1/4}\right) + C$$

$$\frac{2}{3}x^3 - 10x^{1/2} + 12x^{1/4} + C$$

9. Find the **general** expression of $f(x)$ if $f''(x) = 3x^3 + 5x^2 - x + 5$

$$f'(x) = \int 3x^3 + 5x^2 - x + 5 dx$$

$$f'(x) = \frac{3x^4}{4} + \frac{5x^3}{3} - \frac{x^2}{2} + 5x + C$$

$$f(x) = \int \frac{3}{4}x^4 + \frac{5}{3}x^3 - \frac{1}{2}x^2 + 5x + C dx$$

$$f(x) = \frac{3}{4} \left(\frac{x^5}{5}\right) + \frac{5}{3} \left(\frac{x^4}{4}\right) - \frac{1}{2} \left(\frac{x^3}{3}\right) + \frac{5x^2}{2} + Cx + K$$

$$f(x) = \frac{3}{20}x^5 + \frac{5}{12}x^4 - \frac{1}{6}x^3 + \frac{5}{2}x^2 + Cx + K$$

10. Find the **specific** expression of $f(x)$ if $f'(x) = 5x^2 + 9x - 4$, $f(0) = 7$

$$f(x) = \int 5x^2 + 9x - 4 dx$$

$$f(x) = \frac{5x^3}{3} + \frac{9x^2}{2} - 4x + C$$

$$7 = \frac{5}{3}(0)^3 + \frac{9}{2}(0)^2 - 4(0) + C$$

$$C = 7$$

$$f(x) = \frac{5}{3}x^3 + \frac{9}{2}x^2 - 4x + 7$$

4.1, 4.2, 4.6 Formula Sheet:

Summation Formulas:

$$1) \sum_{i=1}^n 1 = n$$

$$2) \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$3) \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$4) \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

$$5) \sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$$

Area using Limit Definition

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (\text{width}) * f(\text{left endpoint} + \text{width} * i)$$

$$\text{width} = \frac{b-a}{n}$$

Trapezoid Area:

$$\text{Area} = \frac{1}{2}w(h_1 + h_2)$$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \csc u \cot u du = -\csc u + C$$