Key

AP Calculus AB 4-1, 4-2, 4-6 Quiz Review WS #2

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}\left[(i+1)^2+3(2i-1)^3\right] = (2+1)^2+3(2(2)-1)^3+(3+1)^2+3(2(3)-1)^3+(4+1)^2+3(2(4)-1)^3$$

$$9+3(27)+16+3(125)+25+(343)(3) = \boxed{1535}$$

2. Use Sigma notation to write the sum: $\frac{5-\sqrt{2}}{1}+\frac{5-\sqrt{4}}{4}+\frac{5-\sqrt{6}}{9}+\frac{5-\sqrt{8}}{16}$

$$\boxed{\sum_{i=1}^{4}\frac{5-\sqrt{2i}}{i^2}}$$

3. Use 3 left rectangles to approximate the area of the region bounded by $f(x) = 1+2x^2$, the x-axis, $x=3$, and $x=7$.

$w = \frac{b-a}{n} \rightarrow \frac{7-3}{3} = \frac{4}{3}$

1 2 3

3 $\;$ $13/3$ $\;$ $17/3$ $\;$ $21/3$

$$\text{Area} \approx \tfrac{4}{3}\cdot f(3)+\tfrac{4}{3}\cdot f(13/3)+\tfrac{4}{3}\cdot f(17/3)$$

$$\tfrac{4}{3}(19)+\tfrac{4}{3}(38.55)+\tfrac{4}{3}(65.22) \approx \boxed{163.696}$$

4. Use the table of values on the right to estimate the below:

x	2	5	6	8	12	13	14
f(x)	1	2	8	3	1	6	5

a. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [5, 13]

3 $\;$ 5

5	6	8	12	13
2	8	3	1	6

$3(8)+5(1) = \boxed{29}$

$f(\;) = 3+2(\;)^2$

b. Use 3 left-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [2, 14]

4 $\;$ 6 $\;$ 2

2	5	6	8	12	13	14
1	2	8	3	1	6	5

$4(1)+6(8)+2(1) = \boxed{54}$

c. Use 2 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 14]

6 $\;$ 2

6	8	12	13	14
8	3	1	6	5

$\text{Area} = \frac{w}{2}[h_1+h_2]$

$\frac{1}{2}w[h_1+h_2]$

$\frac{6}{2}[8+1]+\frac{2}{2}[1+5]$

$3(9)+1(6) = \boxed{33}$

5. Given the region bounded by $f(x) = 3+2x^2$, the x-axis, $x=-2$, and $x=1$. Use the **limit definition** to find the exact area of the region.

$\text{width} = \frac{1-(-2)}{n} = \frac{3}{n}$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\cdot f\left(-2+\frac{3}{n}i\right)$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\cdot\left[3+2\left(-2+\frac{3}{n}i\right)^2\right]$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left[3+2\left(4-\frac{12}{n}i+\frac{9}{n^2}i^2\right)\right]$$

$$\left(-2+\frac{3}{n}i\right)\left(-2+\frac{3}{n}i\right)$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left[3+8-\frac{24}{n}i+\frac{18}{n^2}i^2\right] \rightarrow \sum \frac{33}{n}-\frac{72}{n^2}i+\frac{54}{n^3}i^2$$

$$\sum\frac{33}{n}-\sum\frac{72}{n^2}i+\sum\frac{54}{n^3}i^2$$

$$\lim_{n\to\infty}\frac{33}{n}\boxed{\sum_{i=1}^{n}1}-\frac{72}{n^2}\boxed{\sum_{i=1}^{n}i}+\frac{54}{n^3}\boxed{\sum_{i=1}^{n}i^2}$$

$$\lim_{n\to\infty}\frac{33}{n}(n)-\frac{72}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{54}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n\to\infty}\frac{33n}{n}-\frac{72n^2}{2n^2}+\frac{108n^3}{6n^3} = 33-36+18 = \boxed{15}$$

Find the general antiderivative of g(x). (Find $\int g(x)dx$)

6. $g(x) = 3\cos x - 5\sin x + \csc x \cot x - 3\sqrt{x}$

$\int 3\cos x - 5\sin x + \csc x \cot x - 3x^{1/2}\,dx$

$3(+\sin x) - 5(-\cos x) - \csc x - 3\left(\frac{x^{3/2}}{3/2}\right) + c$

$\boxed{3\sin x + 5\cos x - \csc x - 2x^{3/2} + C}$

7. $g(x) = \frac{2}{3(\sqrt[5]{x})} - 3x^2 - \frac{1}{3e^4}$

$G(x) = \int \frac{2}{3}x^{-1/5} - 3x^2 - \frac{1}{3e^4}dx$

$G(x) = \frac{2}{3}\left(\frac{x^{4/5}}{4/5}\right) - 3\cdot\frac{x^3}{3} - \left(\frac{1}{3e^4}\right)x + c$

$\frac{2}{3}\cdot\frac{5}{4}x^{4/5} - x^3 - \frac{1}{3e^4}x + c$

$\boxed{\frac{5}{6}x^{4/5} - x^3 - \frac{1}{3e^4}x + C}$

8. $g(x) = \frac{2x^3 - 5\sqrt{x} + 3(\sqrt[4]{x})}{x}$

$g(x) = (2x^3 - 5x^{1/2} + 3x^{1/4})x^{-1}$

$g(x) = 2x^2 - 5x^{-1/2} + 3x^{-3/4}$

$\int 2x^2 - 5x^{-1/2} + 3x^{-3/4}dx$

$\frac{2x^3}{3} - 5\left(\frac{x^{1/2}}{1/2}\right) + 3\left(\frac{x^{1/4}}{1/4}\right) + c$

$\boxed{\frac{2}{3}x^3 - 10x^{1/2} + 12x^{1/4} + C}$

9. Find the **general** expression of $f(x)$ if $f''(x) = 3x^3 + 5x^2 - x + 5$

$f'(x) = \int 3x^3 + 5x^2 - x + 5\,dx$

$f'(x) = \frac{3x^4}{4} + \frac{5x^3}{3} - \frac{x^2}{2} + 5x + C$

$f(x) = \int \frac{3}{4}x^4 + \frac{5}{3}x^3 - \frac{1}{2}x^2 + 5x + C\,dx$

$f(x) = \frac{3}{4}\left(\frac{x^5}{5}\right) + \frac{5}{3}\left(\frac{x^4}{4}\right) - \frac{1}{2}\left(\frac{x^3}{3}\right) + \frac{5x^2}{2} + Cx + K$

$\boxed{f(x) = \frac{3}{20}x^5 + \frac{5}{12}x^4 - \frac{1}{6}x^3 + \frac{5}{2}x^2 + Cx + k}$

10. Find the **specific** expression of $f(x)$ if $f'(x) = 5x^2 + 9x - 4$, $f(0) = 7$

$f(x) = \int 5x^2 + 9x - 4\,dx$

$f(x) = \frac{5x^3}{3} + \frac{9x^2}{2} - 4x + C$

$7 = \frac{5}{3}(0)^3 + \frac{9}{2}(0)^2 - 4(0) + c$

$C = 7$

$\boxed{f(x) = \frac{5}{3}x^3 + \frac{9}{2}x^2 - 4x + 7}$

4.1, 4.2, 4.6 Formula Sheet:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} \boldsymbol{ca_i} = c\sum_{i=1}^{n} \boldsymbol{a_i}$

Area using Limit Definition

$$\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(left\ endpoint + width * i)$$

$$\text{width} = \frac{b-a}{n}$$

Trapezoid Area:

$$Area = \frac{1}{2} w(h_1 + h_2)$$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$\int \sin u\, du = -\cos u + C$ $\int \cos u\, du = \sin u + C$

$\int \sec^2 u\, du = \tan u + C$ $\int \sec u \tan u\, du = \sec u + C$

$\int \csc^2 u\, du = -\cot u + C$ $\int \csc u \cot u\, du = -\csc u + C$