Non-AP Calculus   4.1-4.5 Integrals Quiz Review WS #1

Show all appropriate work for full credit

1) $\displaystyle \int \frac{x^2(3-4x^3)}{\sqrt{x}}\,dx$

2) $\displaystyle \int 7x^3(5-3x^4)^6\,dx$

3) $\displaystyle \int \frac{3x}{\sqrt[5]{3x^2-2}}\,dx$

4) $\displaystyle \int \frac{3}{x^2}\csc\left(\frac{2}{x}\right)\cot\left(\frac{2}{x}\right)dx$

5) $\displaystyle \int \frac{\csc^2 x}{\sqrt[3]{\cot x}}\,dx$

6) $\displaystyle \int 4x\sqrt{5-x}\,dx$

7) Find the average value of the function over the given interval: (Show all steps!)

$$f(x) = 2x^3 - 2x + 1, \ [1,3]$$

8) Use Properties of Definite Integrals to evaluate:

$$\int_0^4 f(x)dx = -3 \qquad \int_9^4 f(x)dx = -5$$

a) $\int_4^0 3f(x)dx$

b) $\int_0^9 5f(x)dx$

c) $\int_5^5 2f(x) - g(x)dx$

d) $\int_4^9 -2f(x) + 4 \ dx$

9) Let $\int_{-1}^{-3} g(x)dx = 5$ and $\int_0^1 g(x)dx = -3$

a) If g(x) is even, find $\int_{-1}^3 g(x)dx$

b) If g(x) is odd, find $\int_{-1}^3 g(x)dx$

Show all appropriate work for full credit

Key

1) $\int \dfrac{x^2(3-4x^3)}{\sqrt{x}}\,dx$

$\int (3x^2-4x^5)x^{-1/2}\,dx = \int 3x^{3/2}-4x^{9/2}\,dx$

$\dfrac{3x^{5/2}}{5/2} - 4\cdot\dfrac{x^{11/2}}{11/2}$

$\boxed{\dfrac{6}{5}x^{5/2} - \dfrac{8}{11}x^{11/2}+C}$

$3\cdot\dfrac{2}{5}x^{5/2} - 4\cdot\dfrac{2}{11}x^{11/2}$

2) $\int 7x^3(5-3x^4)^6\,dx$

$u=5-3x^4$

$\dfrac{du}{dx}=-12x^3$

$du=-12x^3\,dx$

$\dfrac{du}{-12x^3}=dx$

$\int 7x^3\cdot u^6\cdot \dfrac{du}{-12x^3}$

$-\dfrac{7}{12}\int u^6\,du = \dfrac{-7}{12}\cdot\dfrac{u^7}{7}+C$

$\boxed{\dfrac{-1}{12}(5-3x^4)^7+C}$

3) $\int \dfrac{3x}{\sqrt[5]{3x^2-2}}\,dx = \int \dfrac{3x}{(3x^2-2)^{1/5}}\,dx$

$\int 3x(3x^2-2)^{-1/5}\,dx$

$u=3x^2-2$

$\dfrac{du}{dx}=6x$

$du=6x\,dx$

$\dfrac{du}{6x}=dx$

$\int 3x\cdot u^{-1/5}\cdot\dfrac{du}{6x}$

$\dfrac{1}{2}\int u^{-1/5}\,du$

$=\dfrac{1}{2}\cdot\dfrac{u^{4/5}}{4/5} = \dfrac{1}{2}\cdot\dfrac{5}{4}u^{4/5}$

$=\boxed{\dfrac{5}{8}(3x^2-2)^{4/5}+C}$

4) $\int \dfrac{3}{x^2}\csc\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)\,dx$

$u=\dfrac{2}{x}=2x^{-1}$

$\dfrac{du}{dx}=-2x^{-2}$

$\dfrac{du}{dx}=\dfrac{-2}{x^2}$

$x^2\,du=-2\,dx$

$\dfrac{x^2\,du}{-2}=dx$

$\int \dfrac{3}{x^2}\csc u\cot u\cdot\dfrac{x^2\,du}{-2}$

$-\dfrac{3}{2}\int \csc u\cot u\,du$

$=-\dfrac{3}{2}\cdot-\csc u+C$

$=\boxed{\dfrac{3}{2}\csc\left(\dfrac{2}{x}\right)+C}$

5) $\int \dfrac{\csc^2 x}{\sqrt[3]{\cot x}}\,dx \qquad \int \dfrac{\csc^2 x}{(\cot x)^{1/3}}\,dx$

$\int \csc^2 x(\cot x)^{-1/3}\,dx$

$u=\cot x$

$\dfrac{du}{dx}=-\csc^2 x$

$du=-\csc^2 x\,dx$

$\dfrac{du}{-\csc^2 x}=dx$

$\int \csc^2 x\cdot u^{-1/3}\cdot\dfrac{du}{-\csc^2 x}$

$-\int u^{-1/3}\,du$

$=-\dfrac{u^{2/3}}{2/3}+C$

$=\boxed{-\dfrac{3}{2}(\cot x)^{2/3}+C}$

6) $\int 4x\sqrt{5-x}\,dx$

$\int 4x(5-x)^{1/2}\,dx$

$u=5-x$

$\dfrac{du}{dx}=-1$

$dx=-du \qquad x=5-u$

$\int 4x\cdot u^{1/2}\cdot-du$

$\int 4(5-u)\cdot u^{1/2}\cdot-du$

$\int -4u^{1/2}(5-u)\,du$

$\int -20u^{1/2}+4u^{3/2}\,du$

$-20\dfrac{u^{3/2}}{3/2}+4\cdot\dfrac{u^{5/2}}{5/2}+C$

$-20\cdot\dfrac{2}{3}u^{3/2}+4\cdot\dfrac{2}{5}u^{5/2}+C$

$\boxed{\dfrac{-40}{3}(5-x)^{3/2}+\dfrac{8}{5}(5-x)^{5/2}+C}$

7) Find the average value of the function over the given interval: (Show all steps!)

$f(x) = 2x^3 - 2x + 1$, $[1,3]$

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

$= \frac{2x^4}{4} - \frac{2x^2}{2} + x = \frac{x^4}{2} - x^2 + x\Big]_1^3$

$f(c) = \frac{1}{3-1}\int_1^3 2x^3 - 2x + 1\,dx$

$= \frac{3^4}{2} - 3^2 + 3 - \left(\frac{1}{2} - 1^2 + 1\right)$

$= \frac{1}{2}\int_1^3 2x^3 - 2x + 1\,dx$

$= \frac{81}{2} - 9 + 3 - \frac{1}{2} = 34$

$f(c) = \frac{1}{2}(34) = \boxed{17}$

8) Use Properties of Definite Integrals to evaluate:

$\int_0^4 f(x)dx = -3$ $\int_9^4 f(x)dx = -5 \to \int_4^9 f(x)dx = 5$

a) $\int_4^0 3f(x)dx$ $\int_4^0 f(x)dx = 3$

$3\left(\int_4^0 f(x)dx\right) = 3(+3) = \boxed{+9}$

b) $\int_0^9 5f(x)dx = 5\left[\int_0^4 f(x)dx + \int_4^9 f(x)dx\right]$

$= 5[-3 + 5] = 5(2) = \boxed{10}$

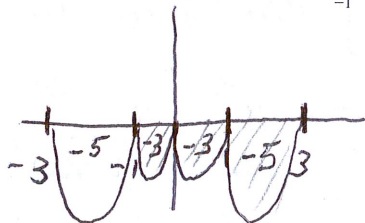c) $\int_5^5 2f(x) - g(x)dx = \boxed{0}$

$\int_5^5 f(x)dx = 0$

d) $\int_4^9 -2f(x) + 4\,dx$

*power rule

$-2\int_4^9 f(x) + \int_4^9 4\,dx \to 4x\Big]_4^9 = 4(9) - 4(4)$

$= 36 - 16 = 20$

$-2(5) + 20 = -10 + 20 = \boxed{10}$

9) Let $\int_{-1}^{-3} g(x)\,dx = 5$ and $\int_0^1 g(x)\,dx = -3$

$\int_{-3}^{-1} g(x)dx = -5$

a)   If g(x) is even, find $\int_{-1}^3 g(x)\,dx = -3 - 3 - 5$

$= \boxed{-11}$

b)   If g(x) is odd, find $\int_{-1}^3 g(x)\,dx = 3 - 3 + 5 = \boxed{5}$