Non-AP Calculus 4.1-4.5 Integrals Quiz Review WS #2

Show all appropriate work for full credit

1) $\int \frac{(1-2x^3)^2}{\sqrt{x}}\,dx$

2) $\int \frac{3x^3}{(9-2x^4)^5}\,dx$

3) $\int \frac{2x^2}{\sqrt{(7-x^3)^5}}\,dx$

4) $\int \frac{3}{\sqrt{x}}\sec^2\left(\sqrt{x}\right)dx$

5) $\int \frac{2\sin x}{\sqrt{(\cos x)^3}}\,dx$

6) $\int 2x\sqrt{6-x}\,dx$

7) Find the average value of the function over the given interval: (Show all steps!)

$f(x) = 3x^2 - 5x + 1$, [-1,4]

8) Use Properties of Definite Integrals to evaluate:

$\int_1^7 f(x)dx = 2 \qquad \int_9^7 f(x)dx = 3$

a) $\int_9^1 2f(x)dx$

b) $\int_7^9 5f(x)dx$

c) $\int_5^5 2f(x) - g(x)dx$

d) $\int_1^9 -f(x) + 2\,dx$

9) Let $\int_0^5 g(x)\,dx = -7$ and $\int_5^3 g(x)\,dx = -3$

a) If g(x) is even, find $\int_{-3}^5 g(x)\,dx$

b) If g(x) is odd, find $\int_{-3}^5 g(x)\,dx$

Non-AP Calculus 4.1-4.5 Integrals Quiz Review WS #2

Key

Show all appropriate work for full credit

1) $\int \frac{(1-2x^3)^2}{\sqrt{x}}\,dx$

$\int \frac{(1-2x^3)(1-2x^3)}{x^{1/2}}\,dx = \int (1-4x^3+4x^6)x^{-1/2}\,dx$

$\int x^{-1/2} - 4x^{5/2} + 4x^{11/2}\,dx$

$\frac{x^{1/2}}{1/2} - 4\cdot\frac{x^{7/2}}{7/2} + \frac{4x^{13/2}}{13/2}$

$2x^{1/2} - 4\cdot\frac{2}{7}x^{7/2} + 4\cdot\frac{2}{13}x^{13/2}$

$= \boxed{2x^{1/2} - \frac{8}{7}x^{7/2} + \frac{8}{13}x^{13/2} + C}$

2) $\int \frac{3x^3}{(9-2x^4)^5}\,dx = \int 3x^3(9-2x^4)^{-5}\,dx$

$u = 9-2x^4$

$\frac{du}{dx} = -8x^3$

$du = -8x^3\,dx$

$\frac{du}{-8x^3} = dx$

$\int 3x^3 \cdot u^{-5} \cdot \frac{du}{-8x^3}$

$= -\frac{3}{8}\int u^{-5}\,du = -\frac{3}{8}\cdot\frac{u^{-4}}{-4} + C$

$= \frac{3}{32}u^{-4} = \boxed{\frac{3}{32(9-2x^4)^4} + C}$

3) $\int \frac{2x^2}{\sqrt{(7-x^3)^5}}\,dx$ $\int 2x^2(7-x^3)^{-5/2}\,dx$

$u = 7-x^3$

$\frac{du}{dx} = -3x^2$

$du = -3x^2\,dx$

$\frac{du}{-3x^2} = dx$

$\int 2x^2 \cdot u^{-5/2} \cdot \frac{du}{-3x^2}$

$-\frac{2}{3}\int u^{-5/2}\,du$

$-\frac{2}{3}\cdot\frac{u^{-3/2}}{-3/2}$

$-\frac{2}{3}\cdot\frac{-2}{3}u^{-3/2} + C$

$\frac{4}{9}u^{-3/2} + C$

$\frac{4}{9u^{3/2}} + C$

$\boxed{\frac{4}{9(7-x^3)^{3/2}} + C}$

4) $\int \frac{3}{\sqrt{x}}\sec^2(\sqrt{x})\,dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{3}{\sqrt{x}}\sec^2 u \cdot 2\sqrt{x}\,du$

$6\int \sec^2 u\,du$

$6\tan u + C$

$= \boxed{6\tan(\sqrt{x}) + C}$

5) $\int \frac{2\sin x}{\sqrt{(\cos x)^3}}\,dx$

$\int 2\sin x(\cos x)^{-3/2}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$\frac{du}{-\sin x} = dx$

$\int 2\sin x \cdot u^{-3/2} \cdot \frac{du}{-\sin x}$

$-2\int u^{-3/2}\,du$

$= -2\cdot\frac{u^{-1/2}}{-1/2}$

$= -2\cdot\frac{-2}{1}u^{-1/2} = 4u^{-1/2} = \frac{4}{u^{1/2}} = \boxed{\frac{4}{(\cos x)^{1/2}} + C}$

6) $\int 2x\sqrt{6-x}\,dx$ $\int 2x(6-x)^{1/2}\,dx$

$u = 6-x$

$\frac{du}{dx} = -1$

$dx = -du$

$x = 6-u$

$\int 2x \cdot u^{1/2} \cdot -du$

$\int 2(6-u)u^{1/2} \cdot -du$

$\int -2u^{1/2}(6-u)\,du$

$\int -12u^{1/2} + 2u^{3/2}\,du$

$-12\cdot\frac{u^{3/2}}{3/2} + 2\cdot\frac{u^{5/2}}{5/2} + C$

$-12\cdot\frac{2}{3}u^{3/2} + 2\cdot\frac{2}{5}u^{5/2} + C$

$\boxed{-8(6-x)^{3/2} + \frac{4}{5}(6-x)^{5/2} + C}$

7) Find the average value of the function over the given interval: (Show all steps!)

$f(x) = 3x^2 - 5x + 1$, [-1,4]

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

$f(c) = \frac{1}{4-(-1)}\int_{-1}^{4} 3x^2 - 5x + 1\,dx$

$= \frac{1}{5}\int_{-1}^{4} 3x^2 - 5x + 1\,dx$

$\frac{3x^3}{3} - \frac{5x^2}{2} + x \Big]_{-1}^{4} = \left(4^3 - \frac{5}{2}(4)^2 + 4\right) - \left((-1)^3 - \frac{5}{2}(-1)^2 - 1\right)$

$64 - 40 + 4 - \left(-1 - \frac{5}{2} - 1\right)$

$28 - (-4.5) = 32.5$

$f(c) = \frac{1}{5}(32.5)$

$f(c) = 6.5$ or $\frac{13}{2}$

8) Use Properties of Definite Integrals to evaluate:

$\int_1^7 f(x)dx = 2 \qquad \int_9^7 f(x)dx = 3 \rightarrow \int_7^9 f(x)dx = -3$

a) $\int_9^1 2f(x)dx = -2\left[\int_1^9 f(x)dx\right]$

$= -2\left[\int_1^7 f(x)dx + \int_7^9 f(x)dx\right] = -2[2 + -3] = -2(-1) = \boxed{2}$

b) $\int_7^9 5f(x)dx = 5\left[\int_7^9 f(x)dx\right]$

$= 5[-3] = \boxed{-15}$

c) $\int_5^5 2f(x) - g(x)dx = \boxed{0}$

d) $\int_1^9 -f(x) + 2\,dx$

$-\int_1^9 f(x)dx + \int_1^9 2dx \rightarrow 2x\Big]_1^9 = 2(9) - 2(1)$

$= 18 - 2$

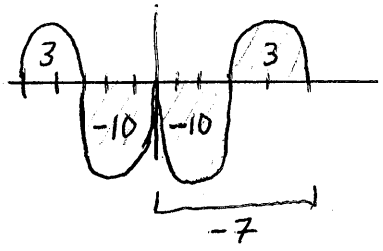
$= 16$

$= -(2-3)$

$= 1 + 16 = \boxed{17}$

9) Let $\int_0^5 g(x)\,dx = -7$ and $\int_5^3 g(x)\,dx = -3$ $\rightarrow \int_3^5 g(x)dx = 3$

a) If g(x) is even, find $\int_{-3}^{5} g(x)\,dx$



$\int_{-3}^{5} g(x)dx = -10 - 10 + 3$

$= \boxed{-17}$

b) If g(x) is odd, find $\int_{-3}^{5} g(x)\,dx = 10 - 10 + 3 = \boxed{3}$