Non-AP Calculus 4.1-4.5 Integrals Quiz Review WS #3

Show all appropriate work for full credit

1) $\int \frac{(3-x^3)^2}{\sqrt{x}}\,dx$

2) $\int \frac{3x^3}{(9-2x^4)^5}\,dx$

3) $\int \frac{2x^2}{\sqrt{(2-2x^3)^7}}\,dx$

4) $\int \frac{5}{\sqrt{x}}\cos\left(\sqrt{x}\right)dx$

5) $\int \frac{2\sin x}{\sqrt{(\cos x)^3}}\,dx$

6) $\int 4x\sqrt{3-x}\,dx$

7) Find the average value of the function over the given interval: (Show all steps!)

$$f(x) = 2x^2 - 2x + 1, \; [-2,3]$$

8) Use Properties of Definite Integrals to evaluate:

$$\int_{-2}^{4} f(x)dx = 2 \qquad \int_{7}^{4} f(x)dx = 3$$

a) $\int_{4}^{-2} 2f(x)dx$

b) $\int_{4}^{-2} 5f(x)dx$

c) $\int_{6}^{6} \frac{(3x-5)^2}{\sqrt{x}}dx$

d) $\int_{-2}^{7} -f(x) + 2\,dx$

9) Let $\int_{5}^{0} g(x)\,dx = -7$ and $\int_{-7}^{0} g(x)\,dx = -3$

a) If g(x) is even, find $\int_{-5}^{7} g(x)\,dx$

b) If g(x) is odd, find $\int_{-7}^{5} g(x)\,dx$

Non-AP Calculus 4.1-4.5 Integrals Quiz Review WS #3

Key

Show all appropriate work for full credit

1) $\int \frac{(3-x^3)^2}{\sqrt{x}}dx = \int \frac{(3-x^3)(3-x^3)}{x^{1/2}}$

$\int (9-6x^3+x^6)x^{-1/2}dx$

$\int 9x^{-1/2}-6x^{5/2}+x^{11/2}dx$

$\frac{9x^{1/2}}{1/2}-\frac{6x^{7/2}}{7/2}+\frac{x^{13/2}}{13/2}$

$18x^{1/2}-6\cdot\frac{2}{7}x^{7/2}+\frac{2}{13}x^{13/2}$

$\boxed{18x^{1/2}-\frac{12}{7}x^{7/2}+\frac{2}{13}x^{13/2}+C}$

2) $\int \frac{3x^3}{(9-2x^4)^5}dx$

$u=9-2x^4$

$\frac{du}{dx}=-8x^3$

$du=-8x^3dx$

$\frac{du}{-8x^3}=dx$

$\int 3x^3\cdot(9-2x^4)^{-5}dx$

$\int 3\cancel{x^3}\cdot u^{-5}\cdot\frac{du}{-8\cancel{x^3}}$

$-\frac{3}{8}\int u^{-5}du$

$-\frac{3}{8}\cdot\frac{u^{-4}}{-4}+C$

$\frac{3}{32}u^{-4}+C$

$\frac{3}{32u^4}+C$

$\boxed{\frac{3}{32(9-2x^4)^4}+C}$

3) $\int \frac{2x^2}{\sqrt{(2-2x^3)^7}}dx$

$\int 2x^2(2-2x^3)^{-7/2}dx$

$u=2-2x^3$

$\frac{du}{dx}=-6x^2$

$du=-6x^2dx$

$\frac{du}{-6x^2}=dx$

$\int 2\cancel{x^2}\cdot u^{-7/2}\cdot\frac{du}{-6\cancel{x^2}}$

$-\frac{1}{3}\int u^{-7/2}du$

$-\frac{1}{3}\cdot\frac{u^{-5/2}}{-5/2}=\frac{1}{3}\cdot\frac{2}{5}u^{-5/2}+C$

$\boxed{\frac{2}{15(2-2x^3)^{5/2}}+C}$

4) $\int \frac{5}{\sqrt{x}}\cos(\sqrt{x})dx$

$u=\sqrt{x}=x^{1/2}$

$\frac{du}{dx}=\frac{1}{2}x^{-1/2}$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$dx=2\sqrt{x}du$

$\int \frac{5}{\cancel{\sqrt{x}}}\cos u\cdot 2\cancel{\sqrt{x}}du$

$10\int \cos u\,du = 10\sin u + C$

$\boxed{10\sin(\sqrt{x})+C}$

5) $\int \frac{2\sin x}{\sqrt{(\cos x)^3}}dx$

$\int 2\sin x(\cos x)^{-3/2}dx$

$u=\cos x$

$\frac{du}{dx}=-\sin x$

$du=-\sin x\,dx$

$\frac{du}{-\sin x}=dx$

$\int 2\cancel{\sin x}\cdot u^{-3/2}\cdot\frac{du}{-\cancel{\sin x}}$

$-2\int u^{-3/2}du$

$-2\cdot\frac{u^{-1/2}}{-1/2}+C$

$-2\cdot-\frac{2}{1}u^{-1/2}+C$

$\boxed{\frac{4}{(\cos x)^{1/2}}+C}$

6) $\int 4x\sqrt{3-x}\,dx$

$\int 4x(3-x)^{1/2}dx$

$u=3-x$

$\frac{du}{dx}=-1$

$dx=-1du$

$\int 4x\cdot u^{1/2}\cdot -1du$

$x=3-u$

$\int 4(3-u)\cdot u^{1/2}\cdot -1du$

$\int -4u^{1/2}(3-u)du$

$\int -12u^{1/2}+4u^{3/2}du$

$-\frac{12u^{3/2}}{3/2}+\frac{4u^{5/2}}{5/2}+C$

$-12\cdot\frac{2}{3}u^{3/2}+4\cdot\frac{2}{5}u^{5/2}+C$

$\boxed{-8(3-x)^{3/2}+\frac{8}{5}(3-x)^{5/2}+C}$

$\boxed{\text{WS \#3}}$

7) Find the average value of the function over the given interval: (Show all steps!)

$f(x) = 2x^2 - 2x + 1$, [-2,3]

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f(c) = \frac{1}{3-(-2)}\int_{-2}^{3} 2x^2-2x+1\,dx$$

$$f(c) = \frac{1}{5}\int_{-2}^{3} 2x^2-2x+1\,dx$$

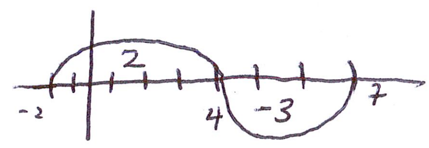
$$\int 2x^2-2x+1\,dx = \frac{2x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 1x\Big]_{-2}^{3}$$

$$\frac{1}{5}\left[\left(\frac{2}{3}(3)^3 - (3)^2 + 3\right) - \left(\frac{2(-2)^3}{3} - (-2)^2 + 1(-2)\right)\right]$$

$$f(c) = \frac{1}{5}\left(\frac{70}{3}\right) = \frac{70}{15} = \boxed{\frac{14}{3}}$$

8) Use Properties of Definite Integrals to evaluate:

$\int_{-2}^{4} f(x)dx = 2$ $\quad \int_{7}^{4} f(x)dx = 3 \rightarrow \int_4^7 f(x)dx = -3$



a) $\int_{4}^{-2} 2f(x)dx$

$$-2\int_{-2}^{4} f(x)dx = -2(2) = \boxed{-4}$$

b) $\int_{4}^{-2} 5f(x)dx$

$$-5\int_{-2}^{4} f(x)dx = -5(2) = -10$$

c) $\int_{6}^{6} \frac{(3x-5)^2}{\sqrt{x}}dx$

$$= \boxed{0}$$

d) $\int_{-2}^{7} -f(x) + 2\,dx$

$$-1\int_{-2}^{7} f(x)dx + \int_{-2}^{7} 2dx \rightarrow 2x\Big]_{-2}^{7} = 2(7)-2(-2) = 14+4 = 18$$

$$-1\left[\int_{-2}^{4} f(x)dx + \int_4^7 f(x)dx\right]$$

$$-1[2+-3]$$

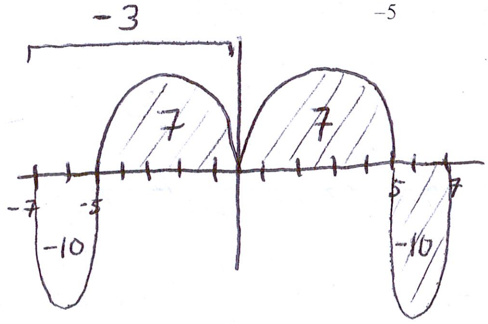
$$1 + 18 = \boxed{19}$$

9) Let $\int_{5}^{0} g(x)\,dx = -7$ and $\int_{-7}^{0} g(x)\,dx = -3$ $\quad \int_0^5 g(x)dx = 7$

a) If g(x) is even, find $\int_{-5}^{7} g(x)\,dx$

*graph region with smaller interval first.

$\int_0^5 g(x)dx = 7$



$$\int_{-5}^{7} f(x)dx = 7+7-10 = \boxed{4}$$

b) If g(x) is odd, find $\int_{-7}^{5} g(x)\,dx = 4-7+7 = \boxed{4}$