**Non-AP Calculus** **Ch. 4.1** **Find the Indefinite Integral: Classwork Worksheet**

[redacted] 13) $\int (x^5 + 1)\,dx$

14. $\int (8x^3 - 9x^2 + 4)\,dx$

[redacted] 15) $\int (x^{3/2} + 2x + 1)\,dx$

16. $\int \left(\sqrt{x} + \dfrac{1}{2\sqrt{x}}\right) dx$

[redacted] 19) $\int \dfrac{1}{x^5}\,dx$

20. $\int \dfrac{3}{x^7}\,dx$

[redacted] 21) $\int \dfrac{x + 6}{\sqrt{x}}\,dx$

22. $\int \dfrac{x^4 - 3x^2 + 5}{x^4}\,dx$

23) $\displaystyle\int (x + 1)(3x - 2)\, dx$

**24.** $\displaystyle\int (4t^2 + 3)^2\, dt$

25) $\displaystyle\int (5 \cos x + 4 \sin x)\, dx$

**26.** $\displaystyle\int (t^2 - \cos t)\, dt$

27) $\displaystyle\int (1 - \csc t \cot t)\, dt$

**28.** $\displaystyle\int (\theta^2 + \sec^2 \theta)\, d\theta$

29) $\displaystyle\int (\sec^2 \theta - \sin \theta)\, d\theta$

**30.** $\displaystyle\int \sec y (\tan y - \sec y)\, dy$

**Finding a Particular Solution** **In Exercises 35–42, find the particular solution that satisfies the differential equation and the initial condition.**

**35.** $f'(x) = 6x,\ f(0) = 8$

**36.** $g'(x) = 4x^2,\ g(-1) = 3$

**38.** $f'(s) = 10s - 12s^3,\ f(3) = 2$

**Finding a Particular Solution** In Exercises 35–42, find the particular solution that satisfies the differential equation and the initial condition.

**39.** $f''(x) = 2, \; f'(2) = 5, \; f(2) = 10$

**40.** $f''(x) = x^2, \; f'(0) = 8, \; f(0) = 4$

**41.** $f''(x) = x^{-3/2}, \; f'(4) = 2, \; f(0) = 0$

Key

13) $\int (x^5 + 1)\, dx$

$\boxed{\frac{x^6}{6} + x + C}$

14. $\int (8x^3 - 9x^2 + 4)\, dx$

$\frac{8x^4}{4} - \frac{9x^3}{3} + 4x + C = \boxed{2x^4 - 3x^3 + 4x + C}$

15) $\int (x^{3/2} + 2x + 1)\, dx$

$\frac{x^{5/2}}{5/2} + \frac{2x^2}{2} + x + C$

$\boxed{\frac{2}{5}x^{5/2} + x^2 + x + C}$

16. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

$= \int x^{1/2} + \frac{1}{2}\int x^{-1/2}\, dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{3}x^{3/2} + x^{1/2} + C}$

19) $\int \frac{1}{x^5}\, dx$

$\int x^{-5}\, dx = \frac{x^{-4}}{-4} + C$

$= \boxed{-\frac{1}{4x^4} + C}$

20. $\int \frac{3}{x^7}\, dx$

$\int 3x^{-7}\, dx = \frac{3x^{-6}}{-6} + C$

$= -\frac{1}{2x^6} + C$

$= \boxed{-\frac{1}{2x^6} + C}$

21) $\int \frac{x + 6}{\sqrt{x}}\, dx$

$\int \frac{x + 6}{x^{1/2}}\, dx$

$\int (x+6)x^{-1/2}\, dx$

$\int x^{1/2} + 6x^{-1/2}\, dx$

$\frac{x^{3/2}}{3/2} + \frac{6x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{3}x^{3/2} + 12x^{1/2} + C}$

22. $\int \frac{x^4 - 3x^2 + 5}{x^4}\, dx$

$\int (x^4 - 3x^2 + 5)x^{-4}\, dx$

$\int 1 - 3x^{-2} + 5x^{-4}\, dx$

$x - \frac{3x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C$

$\boxed{x + \frac{3}{x} - \frac{5}{3x^3} + C}$

23) $\int (x + 1)(3x - 2)\, dx$

$\int 3x^2 - 2x + 3x - 2\, dx$

$\int 3x^2 + x - 2\, dx$

$\frac{3x^3}{3} + \frac{x^2}{2} - 2x + c$

$\boxed{x^3 + \frac{1}{2}x^2 - 2x + c}$

**24.** $\int (4t^2 + 3)^2\, dt$

$\int (4t^2+3)(4t^2+3)\, dt$

$\int 16t^4 + 12t^2 + 12t^2 + 9\, dt$

$\int 16t^4 + 24t^2 + 9\, dt$

$\frac{16t^5}{5} + \frac{24t^3}{3} + 9t + C$

$\boxed{\frac{16t^5}{5} + 8t^3 + 9t + C}$

25) $\int (5 \cos x + 4 \sin x)\, dx$

$\boxed{5\sin x - 4\cos x + C}$

**26.** $\int (t^2 - \cos t)\, dt$

$\boxed{\frac{t^3}{3} - \sin t + C}$

27) $\int (1 - \csc t \cot t)\, dt$

$\boxed{t + \csc t + C}$

**28.** $\int (\theta^2 + \sec^2 \theta)\, d\theta$

$\boxed{\frac{\theta^3}{3} + \tan\theta + C}$

29) $\int (\sec^2 \theta - \sin \theta)\, d\theta$

$\boxed{\tan\theta + \cos\theta + C}$

**30.** $\int \sec y (\tan y - \sec y)\, dy$

$\int \sec y \tan y - \sec^2 y\, dy$

$\boxed{\sec y - \tan y + C}$

**Finding a Particular Solution** In Exercises 35–42, find the particular solution that satisfies the differential equation and the initial condition.

**35.** $f'(x) = 6x,\ f(0) = 8$

$f(x) = \int 6x\,dx$

$f(x) = \frac{6x^2}{2} + C$

$f(x) = 3x^2 + C$ ← *plug in (0, 8)

$8 = 3(0)^2 + C$

$8 = C$

$\boxed{f(x) = 3x^2 + 8}$

**36.** $g'(x) = 4x^2,\ g(-1) = 3$

$g(x) = \int 4x^2\,dx$

$g(x) = \frac{4x^3}{3} + C$ ← *plug in (-1, 3)

$3 = \frac{4}{3}(-1)^3 + C$

$3 = -\frac{4}{3} + C$

$3 + \frac{4}{3} = C,\ C = \frac{13}{3}$

$\boxed{g(x) = \frac{4}{3}x^3 + \frac{13}{3}}$

**38.** $f'(s) = 10s - 12s^3,\ f(3) = 2$

$f(s) = \int 10s - 12s^3\,ds$

$f(s) = \frac{10s^2}{2} - \frac{12s^4}{4} + C$

$f(s) = 5s^2 - 3s^4 + C$

↑ plug in (3, 2)

$2 = 5(3)^2 - 3(3)^4 + C$

$2 = 45 - 243 + C$

$2 = -198 + C$

$200 = C$

$\boxed{f(s) = 5s^2 - 3s^4 + 200}$

**Finding a Particular Solution** In Exercises 35–42, find the particular solution that satisfies the differential equation and the initial condition.

**39.** $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$

$f'(x) = \int 2\,dx$

$f'(x) = 2x + C_1$ plug in $f'(2)=5$

$5 = 2(2) + C_1$

$5 = 4 + C_1$

$\boxed{1 = C_1}$

$f'(x) = 2x + 1$

$f(x) = \int 2x + 1\,dx$

$f(x) = \frac{2x^2}{2} + x + C$

$f(x) = x^2 + x + C$ *plug in $(2,10)$

$f(x) = x^2 + x + C_2$

$10 = 2^2 + 2 + C_2$

$10 = 6 + C_2$

$\boxed{4 = C_2}$

$\boxed{f(x) = x^2 + x + 4}$

**40.** $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

$f'(x) = \int x^2\,dx$

$f'(x) = \frac{x^3}{3} + C_1$ *plug in $f'(0)=8$

$8 = \frac{0^3}{3} + C_1$

$8 = C_1$

$f'(x) = \frac{x^3}{3} + 8$

$f(x) = \int \frac{x^3}{3} + 8$

$f(x) = \frac{x^4}{12} + 8x + C_2$ plug in $(0,4)$

$4 = \frac{0^4}{12} + 8(0) + C_2$

$4 = C_2$

$\boxed{f(x) = \frac{x^4}{12} + 8x + 4}$

**41.** $f''(x) = x^{-3/2},\ f'(4) = 2,\ f(0) = 0$

$f'(x) = \int x^{-3/2}\,dx$

$f'(x) = \frac{x^{-1/2}}{-1/2} + C_1$

$f'(x) = -2x^{-1/2} + C_1$ plug in $f'(4)=2$

$2 = -2(4)^{-1/2} + C_1$

$2 = \frac{-2}{4^{1/2}} + C_1$

$2 = \frac{-2}{2} + C_1$

$2 + 1 = C_1$

$3 = C_1$

$f'(x) = -2x^{-1/2} + 3$

$f(x) = \int -2x^{-1/2} + 3\,dx$

$f(x) = \frac{-2x^{1/2}}{1/2} + 3x + C_2$ plug in $(0,0)$

$f(x) = -4x^{1/2} + 3x + C_2$

$0 = 0 + 0 + C_2$

$\boxed{C_2 = 0}$

$\boxed{f(x) = -4x^{1/2} + 3x}$