A.P. Calculus AB **4.1 Morning Review**

Find the general antiderivative of $g(x)$.

1. $g(x) = x(2x-1)^2$

2. $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

3. $g(x) = \frac{x^3 - 2\sqrt{x} + \sqrt[4]{x}}{\sqrt{x}}$

4. Find the **general** expression of $f(x)$ if $f''(x) = 2x^3 + 3x^2 + x - 1$

5. Find the **specific** expression of $f(x)$ if $f''(x) = 12x^2 + 18x - 4$, $f'(-1) = 9$, and $f(1) = 3$

A.P. Calculus AB **4.1 Morning Review** Key

Find the general antiderivative of $g(x)$.

1. $g(x) = x(2x-1)^2$

$\int x(2x-1)^2\,dx$

$\int x(4x^2-2x+1)\,dx$

$\int 4x^3-2x^2+x\,dx$

$\frac{4x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + C$

$\boxed{x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} + C}$

2. $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

$\int 4x^{-1/3} - x^{1/2} + 3x^2 - \frac{1}{3}x^{-4}\,dx$

$\frac{4x^{2/3}}{2/3} - \frac{x^{3/2}}{3/2} + \frac{3x^3}{3} - \frac{1}{3}\left(\frac{x^{-3}}{-3}\right) + C$

$4(3/2)x^{2/3} - \frac{2}{3}x^{3/2} + x^3 + \frac{1}{9}\left(x^{-3}\right) + C$

$\boxed{6x^{2/3} - \frac{2}{3}x^{3/2} + x^3 + \frac{1}{9x^3} + C}$

3. $g(x) = \frac{x^3 - 2\sqrt{x} + \sqrt[4]{x}}{\sqrt{x}}$

$\int \left(x^3 - 2x^{1/2} + x^{1/4}\right)x^{-1/2}\,dx$

$\int x^{5/2} - 2 + x^{-1/4}\,dx$

$\frac{x^{7/2}}{7/2} - 2x + \frac{x^{3/4}}{3/4} + C$

$\boxed{\frac{2}{7}x^{7/2} - 2x + \frac{4}{3}x^{3/4} + C}$

4. Find the **general** expression of $f(x)$ if $f''(x) = 2x^3 + 3x^2 + x - 1$

$f'(x) = \int 2x^3 + 3x^2 + x - 1\,dx$

$f'(x) = \frac{2x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2} - x + C_1$

$f'(x) = \frac{1}{2}x^4 + x^3 + \frac{1}{2}x^2 - x + C_1$

$f(x) = \int \frac{1}{2}x^4 + x^3 + \frac{1}{2}x^2 - x + C_1\,dx$

$f(x) = \frac{1}{2}\left(\frac{x^5}{5}\right) + \frac{x^4}{4} + \frac{1}{2}\left(\frac{x^3}{3}\right) - \frac{x^2}{2} + C_1x + C_2$

$\boxed{f(x) = \frac{x^5}{10} + \frac{x^4}{4} + \frac{1}{6}x^3 - \frac{x^2}{2} + C_1x + C_2}$

5. Find the **specific** expression of $f(x)$ if $f''(x) = 12x^2 + 18x - 4$, $f'(-1) = 9$, and $f(1) = 3$

$f'(x) = \int 12x^2 + 18x - 4\,dx$

$f'(x) = \frac{12x^3}{3} + \frac{18x^2}{2} - 4x + C_1$

$f'(x) = 4x^3 + 9x^2 - 4x + C_1$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C_1$

$9 = -4 + 9 + 4 + C_1$, $C_1 = 0$

$f'(x) = 4x^3 + 9x^2 - 4x + 0$

$f(x) = \int 4x^3 + 9x^2 - 4x\,dx$

$f(x) = \frac{4x^4}{4} + \frac{9x^3}{3} - \frac{4x^2}{2} + C_2$

$f(x) = x^4 + 3x^3 - 2x^2 + C_2$

$3 = 1^4 + 3(1)^3 - 2(1)^2 + C_2$

$3 = 1 + 3 - 2 + C_2$

$1 = C_2$

$\boxed{f(x) = x^4 + 3x^3 - 2x^2 + 1}$