Non-AP Calculus 4.1 Quiz Review WS #2

Find the antiderivative

1. $\int 5x^3 + 3\sqrt{x} + \frac{2}{\sqrt[3]{x}} - \frac{\sqrt{x^7}}{2} + \frac{9}{x^2} - 2(x+1)\,dx$

2. $\int 5\sin x - 12\csc^2 x\,dx$

3. $\int \frac{3x^2 + \sqrt[3]{x} - 2\sqrt{x} - 1}{4\sqrt{x}}\,dx$

4. $\int 1-(2x-5)(x^2+1)\,dx$

5. Find the most general expression of $f(x)$ if $f''(x) = 3x^2 + 2x - 7$

6. Find the specific expression of $f(x)$ if $f''(x) = 6x^3 + 2x^2 + 3$, $f'(0) = -5$, and $f(1) = 2$

Key

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Find the antiderivative

1. $\int 5x^3 + 3\sqrt{x} + \frac{2}{\sqrt[3]{x}} - \frac{\sqrt{x^7}}{2} + \frac{9}{x^2} - 2(x+1)\,dx$

$\int 5x^3 + 3x^{1/2} + 2x^{-1/3} - \frac{1}{2}x^{7/2} + 9x^{-2} - 2x - 2\,dx$

$5\left(\frac{x^4}{4}\right) + 3\left(\frac{x^{3/2}}{3/2}\right) + 2\left(\frac{x^{2/3}}{2/3}\right) - \frac{1}{2}\left(\frac{x^{9/2}}{9/2}\right) + \frac{9x^{-1}}{-1} - \frac{2x^2}{2} - 2x + C$

$\frac{5}{4}x^4 + 3\cdot\frac{2}{3}x^{3/2} + 2\cdot\frac{3}{2}x^{2/3} - \frac{1}{2}\cdot\frac{2}{9}x^{9/2} - \frac{9}{x} - x^2 - 2x + C$

$\boxed{\frac{5}{4}x^4 + 2x^{3/2} + 3x^{2/3} - \frac{1}{9}x^{9/2} - \frac{9}{x} - x^2 - 2x + C}$

2. $\int 5\sin x - 12\csc^2 x\,dx$

$= 5(-\cos x) - 12(-\cot x) + C \quad = \boxed{-5\cos x + 12\cot x + C}$

3. $\int \frac{3x^2 + \sqrt[3]{x} - 2\sqrt{x} - 1}{4\sqrt{x}}\,dx$

$\frac{1}{4}\int (3x^2 + x^{1/3} - 2x^{1/2} - 1)x^{-1/2}\,dx$

$\frac{1}{4}\int 3x^{4\frac{1}{2}-\frac{1}{2}} + x^{\frac{2}{6}-\frac{3}{6}} - 2x^{\frac{1}{2}-\frac{1}{2}} - 1x^{-1/2}\,dx$

$\frac{1}{4}\int 3x^{3/2} + x^{-1/6} - 2 - 1x^{-1/2}\,dx$

$\int \frac{3}{4}x^{3/2} + \frac{1}{4}x^{-1/6} - \frac{1}{2} - \frac{1}{4}x^{-1/2}\,dx$ (+2/2, +6/6, +2/2)

$\frac{3}{4}\left(\frac{x^{5/2}}{5/2}\right) + \frac{1}{4}\left(\frac{x^{5/6}}{5/6}\right) - \frac{1}{2}x - \frac{1}{4}\left(\frac{x^{1/2}}{1/2}\right) + C$

$\frac{3}{4}\cdot\frac{2}{5}x^{5/2} + \frac{1}{4}\cdot\frac{6}{5}x^{5/6} - \frac{1}{2}x - \frac{1}{4}\cdot\frac{2}{1}x^{1/2} + C$

$\boxed{\frac{3}{10}x^{5/2} + \frac{3}{10}x^{5/6} - \frac{1}{2}x - \frac{1}{2}x^{1/2} + C}$

4. $\int 1-(2x-5)(x^2+1)\,dx$

$\int 1-(2x^3+2x-5x^2-5)\,dx$

$\int 1-2x^3-2x+5x^2+5\,dx$

$\int -2x^3+5x^2-2x+6\,dx$

$-\frac{2x^4}{4}+\frac{5x^3}{3}-\frac{2x^2}{2}+6x+C$

$\boxed{-\frac{1}{2}x^4+\frac{5}{3}x^3-x^2+6x+C}$

5. Find the most general expression of $f(x)$ if $f''(x) = 3x^2 + 2x - 7$

$f'(x)=\int 3x^2+2x-7\,dx$

$f'(x)=\frac{3x^3}{3}+\frac{2x^2}{2}-7x+C$

$f'(x)=x^3+x^2-7x+C$

$f(x)=\int x^3+x^2-7x+C\,dx$

$\boxed{f(x)=\frac{x^4}{4}+\frac{x^3}{3}-\frac{7x^2}{2}+Cx+K}$ OR $\boxed{f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{7}{2}x^2+Cx+K}$

6. Find the specific expression of $f(x)$ if $f''(x) = 6x^3 + 2x^2 + 3$, $f'(0) = -5$, and $f(1) = 2$

$f'(x)=\int 6x^3+2x^2+3\,dx$

$f'(x)=\frac{6x^4}{4}+\frac{2x^3}{3}+3x+C$

$f'(x)=\frac{3}{2}x^4+\frac{2}{3}x^3+3x+C$ ($f'(0)=-5$)

$-5=\frac{3}{2}(0)+\frac{2}{3}(0)+3(0)+C$

$-5=C$

$f'(x)=\frac{3}{2}x^4+\frac{2}{3}x^3+3x-5$

$f(x)=\int \frac{3}{2}x^4+\frac{2}{3}x^3+3x-5\,dx$

$f(x)=\frac{3}{2}\cdot\frac{x^5}{5}+\frac{2}{3}\cdot\frac{x^4}{4}+\frac{3x^2}{2}-5x+K$ ($f(1)=2$)

$f(x)=\frac{3}{10}x^5+\frac{1}{6}x^4+\frac{3}{2}x^2-5x+K$

$2=\frac{3}{10}+\frac{1}{6}+\frac{3}{2}-5+K$

$2=-\frac{91}{30}+K$

$\frac{151}{30}=K$

$\boxed{f(x)=\frac{3}{10}x^5+\frac{1}{6}x^4+\frac{3}{2}x^2-5x+\frac{151}{30}}$