Calculus 4.1a Notes Antiderivative Formulas

$If\ f(x) = x^2, what\ is\ f'(x)?$

$Using\ Power\ Rule,\ \frac{d}{dx}u^n = n * u^{n-1}, we\ know\ that\ f'(x) = 2x$

To put the steps for Derivatives Power rule into words:

1) Bring exponent down in front of variable and _______________

2) _______________ exponent by 1

$If\ f'(x) = 2x, what\ steps\ can\ we\ take\ to\ find\ f(x)?$

We can "undo" the previous derivative steps:

1) _______________ 1 to the exponent

2) _______________ by the new exponent

Power Rule for Integration:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Antidifferentiation Notation:

NOTATION: $\int 2x\,dx = x^2 + C$

Integral integrand identifies the independent variable constant of integration

Consider the below functions:

$f(x) = x^2 + 5$
$f(x) = x^2 - 13$
$f(x) = x^2 + 126$

Since we can add a constant to any of these functions and still result in the same derivative, the **antiderivative** of a function will be in the form of $f(x) + C$ to show the family of functions that share the same derivative.

The process of integration is called **antidifferentiation** or taking the indefinite integral.

The indefinite integral results in a function.

The definite integral results in a number.

Integration Formulas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$
2. $\int a\, dx = ax + C$
3. $\int \frac{1}{u}\, du = \ln|u| + C$

Important: The derivative and integral are Inverse operations of each other.

4) $\int f'(x)dx = f(x) + C$

5) $\frac{d}{dx}[\int f(x)\, dx] = f(x)$

Recall Power Rule Conditions:

1) Rewrite as rational exponents 2)All variables in numerator 3) Expand expression fully

Class Examples:

1. $\int 7x\, dx =$

2. $\int 7x^3 dx =$

3. $\int 2x + 3x^2 - 5x^4\, dx =$

4. $\int (3x-1)^2 dx =$

5. $\int \frac{x+1}{\sqrt{x}} dx =$

6. $\int \frac{3}{y\sqrt{y}} dy =$

7. $\int \frac{3\sqrt{x}(1-x)^2}{\sqrt[3]{x}} dx =$

Calculus **4.1b Notes** **Trig Integrals and Differential Equations**

Review Derivative Trig Rules:

1) $\frac{d}{dx}\sin u =$

3) $\frac{d}{dx}\cos u =$

2) $\frac{d}{dx}\tan u =$

4) $\frac{d}{dx}\cot u =$

5) $\frac{d}{dx}\sec u =$

6) $\frac{d}{dx}\csc u =$

Integral Trig Rules:

1) $\int \sin u\, du =$

2) $\int \cos u\, du =$

3) $\int \sec^2 u\, du =$

4) $\int \csc^2 u\, du =$

5) $\int \sec u \tan u\, du =$

6) $\int \csc u \cot u\, du =$

Classwork Examples:

1. $\int \frac{\tan x}{\cos x} - \sin x \; dx$

2. $\int \frac{\sin x}{\cos^2 x} dx$

3. $\int (1 + \cot^2 x) dx$

Differential Equations: These are simply equations that involve derivatives.

Steps for solving Differential equations:

1. Rewrite y′ as $\frac{dy}{dx}$
2. Separate variables on either side of equation
3. Take the integral of both sides

Solve for C if finding a specific solution/equation to the differential equation

Example 3: Suppose y′ = 2. Solve for y.

Example 4: Solve this General Differential equation. $\frac{dy}{dx} = x^3$

Example 5: Solve this Specific differential equation: y′ = 3x – 4 and the point (4, 10) is on the graph of y.

4.16 (continued) More diff. equation examples

$\boxed{\text{Ex. 6}}$ Suppose $f''(x) = 6x + 4$, $f'(0) = 3$, and $f(1) = 5$.
Find $f(x)$.

* To help distinguish the constants of integration, use "+C" for the first constant and use "+K" for the second constant of integration.

$\boxed{\text{Ex. 7}}$ Given $g''(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

Key

4-1 Review

Find the most general antiderivative of $h(x)$.

1. $h(x)=5x^4-5x^2+\pi+\frac{1}{2\sqrt{x}}+\frac{1}{3x^3}$ $=5x^4-5x^2+\pi+\frac{x^{-1/2}}{2}+\frac{1}{3}x^{-3}$

$$H(x)=\int h(x)\,dx=\frac{5x^5}{5}-\frac{5x^3}{3}+\pi x+\frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right)+\frac{1}{3}\left(\frac{x^{-2}}{-2}\right)+C=\boxed{x^5-\frac{5x^3}{3}+\pi x+\sqrt{x}-\frac{1}{6x^2}+C}$$

2. $h(x)=\frac{-8}{\sqrt[3]{x}}-2\csc^2 x$ $=-8x^{-1/3}-2\csc^2 x$

$$\int h(x)\,dx=-8\left(\frac{x^{2/3}}{2/3}\right)-2(-\cot x)$$

$$=-8\left(\frac{3}{2}\right)(x^{2/3})+2\cot x+C \quad = \boxed{-12x^{2/3}+2\cot x+C}$$

3. $h(x)=\frac{7}{\sqrt[5]{x}}-\frac{\sqrt[4]{x}}{13}+10$ $=7x^{-1/5}-\frac{1}{13}x^{1/4}+10$

$$\int h(x)\,dx=\frac{7x^{4/5}}{4/5}-\frac{1}{13}\left(\frac{x^{5/4}}{5/4}\right)+10x+C=$$

$$7\left(\frac{5}{4}\right)x^{4/5}-\frac{1}{13}\left(\frac{4}{5}\right)x^{5/4}+10x+C$$

$$\boxed{\frac{35}{4}x^{4/5}-\frac{4}{65}x^{5/4}+10x+C}$$

4. $h(x)=\frac{2x-6x^3+x^2}{\sqrt[3]{x}}$ $=\frac{2x^1}{x^{1/3}}-\frac{6x^3}{x^{1/3}}+\frac{x^2}{x^{1/3}}=2x^{2/3}-6x^{8/3}+x^{5/3}$

$$\int h(x)\,dx=2\left(\frac{x^{5/3}}{5/3}\right)-6\left(\frac{x^{11/3}}{11/3}\right)+\frac{x^{8/3}}{8/3}+C$$

$$=2\left(\frac{3}{5}\right)x^{5/3}-6\left(\frac{3}{11}\right)x^{11/3}+\frac{3}{8}x^{8/3}+C$$

$$\boxed{\frac{6x^{5/3}}{5}-\frac{18x^{11/3}}{11}+\frac{3x^{8/3}}{8}+C}$$

5. $h(x)=-2\cos x+5\sin x-5\csc x\cot x$

$$\int h(x)\,dx=-2(\sin x)+5(-\cos x)-5(-\csc x)+C$$

$$=\boxed{-2\sin x-5\cos x+5\csc x+C}$$

6. Find the most general expression of $f(x)$ if $f''(x)=9x^2-5x+2$.

$$f'(x)=\frac{9x^3}{3}-\frac{5x^2}{2}+2x+c=3x^3-\frac{5x^2}{2}+2x+c$$

$$f(x)=\frac{3x^4}{4}-\frac{5}{2}\left(\frac{x^3}{3}\right)+\frac{2x^2}{2}+cx+C_2$$

$$=\boxed{\frac{3}{4}x^4-\frac{5x^3}{6}+x^2+cx+C_2}$$

7. Find the most general expression of $f(x)$ if $f''(x)=4x^3-5x^2+3x-6$.

$$f'(x)=\frac{4x^4}{4}-\frac{5x^3}{3}+\frac{3x^2}{2}-6x+c$$

$$=x^4-\frac{5x^3}{3}+\frac{3x^2}{2}-6x+c$$

$$f(x)=\frac{x^5}{5}-\frac{5}{3}\left(\frac{x^4}{4}\right)+\frac{3}{2}\left(\frac{x^3}{3}\right)-\frac{6x^2}{2}+cx+C_2$$

$$\boxed{f(x)=\frac{x^5}{5}-\frac{5x^4}{12}+\frac{x^3}{2}-3x^2+cx+C_2}$$

8. Find the specific expression of $f(x)$ if $f(x)=\int g(x)dx$, $g(x)=3x^2-4x$, and $f(-1)=2$

$$\int g(x)\,dx=\int 3x^2-4x\,dx=\frac{3x^3}{3}-\frac{4x^2}{2}+c\rightarrow f(x)=x^3-2x^2+c$$

$$f(-1)=(-1)^3-2(-1)^2+c$$

$$2=-1-2+c$$

$$2+3=c$$

$$5=c$$

$$\boxed{f(x)=x^3-2x^2+5}$$

9. Find the specific expression of $f(x)$ if $f''(x)=36x^2-6$, $f'(-1)=3$, and $f(1)=9$.

$$f'(x)=\frac{36x^3}{3}-6x+c=12x^3-6x+c_1$$

$$f'(-1)=12(-1)^3-6(-1)+c_1$$

$$3=-12+6+c_1$$

$$3=-6+c_1$$

$$9=c_1$$

$$f'(x)=12x^3-6x+9$$

$$f(x)=\frac{12x^4}{4}-\frac{6x^2}{2}+9x+C_2$$

$$f(x)=3x^4-3x^2+9x+C_2$$

$$f(1)=3(1)^4-3(1)^2+9(1)+C_2$$

$$9=3-3+9+C_2$$

$$9=0+9+C_2$$

$$0=C_2$$

$$\boxed{f(x)=3x^4-3x^2+9x}$$

Calculus **4-1 Review** WS #1

Find the most general antiderivative of $h(x)$.

1. $h(x)=5x^4-5x^2+\pi+\dfrac{1}{2\sqrt{x}}+\dfrac{1}{3x^3}$

2. $h(x)=\dfrac{-8}{\sqrt[3]{x}}-2\csc^2 x$

3. $h(x)=\dfrac{7}{\sqrt[5]{x}}-\dfrac{\sqrt[4]{x}}{13}+10$

4. $h(x)=\dfrac{2x-6x^3+x^2}{\sqrt[3]{x}}$

5. $h(x)=-2\cos x+5\sin x-5\csc x\cot x$

6. Find the most general expression of $f(x)$ if $f''(x) = 9x^2 - 5x + 2$.

7. Find the most general expression of $f(x)$ if $f''(x) = 4x^3 - 5x^2 + 3x - 6$.

8. Find the specific expression of $f(x)$ if $f(x) = \int g(x)dx$, $g(x) = 3x^2 - 4x$, and $f(-1) = 2$

9. Find the specific expression of $f(x)$ if $f''(x) = 36x^2 - 6$, $f'(-1) = 3$, and $f(1) = 9$.

Key

Calculus 4.1a Notes Antiderivative Formulas

If $f(x) = x^2$, *what is* $f'(x)$?

Using Power Rule, $\frac{d}{dx}u^n = n * u^{n-1}$, *we know that* $f'(x) = 2x$

To put the steps for Derivatives Power rule into words:

1) Bring exponent down in front of variable and multiply
2) subtract exponent by 1

If $f'(x) = 2x$, *what steps can we take to find* $f(x)$? $f(x) = \frac{2x^{1+1}}{2} = x^2$

We can "undo" the previous derivative steps:

1) Add 1 to the exponent
2) Divide by the new exponent

Power Rule for Integration:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Antidifferentiation Notation:

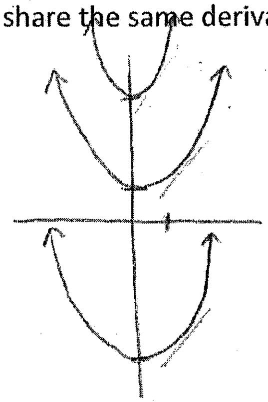
NOTATION: $\int 2x\,dx = x^2 + C$

Integral integrand identifies the independent variable constant of integration

Consider the below functions:

$$\left.\begin{aligned} f(x) &= x^2 + 5 \\ f(x) &= x^2 - 13 \\ f(x) &= x^2 + 126 \end{aligned}\right\} f'(x) = 2x$$

Since we can add a constant to any of these functions and still result in the same derivative, the **antiderivative** of a function will be in the form of $f(x) + C$ to show the family of functions that share the same derivative.



The process of integration is called **antidifferentiation** or taking the indefinite integral.

The indefinite integral results in a function.

The definite integral results in a number.

Integration Formulas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$

2. $\int a\, dx = ax + C$

3. $\int \frac{1}{u}\, du = \ln|u| + C$

Important: The derivative and integral are Inverse operations of each other.

4) $\int f'(x)dx = f(x) + C$

5) $\frac{d}{dx}[\int f(x)\, dx] = f(x)$

Recall Power Rule Conditions:

1) Rewrite as rational exponents 2)All variables in numerator 3) Expand expression fully

Class Examples:

1. $\int 7x\, dx = \boxed{\frac{7x^2}{2} + C}$

1b) $\int 7 dx = \boxed{7x + C}$

2. $\int 7x^3 dx = \boxed{\frac{7x^4}{4} + C}$

3. $\int 2x + 3x^2 - 5x^4\, dx =$

$\frac{2x^2}{2} + \frac{3x^3}{3} - \frac{5x^5}{5} + C$

$\boxed{x^2 + x^3 - x^5 + C}$

4. $\int (3x-1)^2 dx = \int (3x-1)(3x-1)\,dx$

$\int 9x^2 - 6x + 1\, dx$

$\frac{9x^3}{3} - \frac{6x^2}{2} + x + C$

$\boxed{3x^3 - 3x^2 + x + C}$

5. $\int \frac{x+1}{\sqrt{x}} dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\, dx$

$\int x^{1/2} + x^{-1/2} dx$

$\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \boxed{\frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

6. $\int \frac{3}{y\sqrt{y}} dy = \int \frac{3}{y \cdot y^{1/2}} dy = \int \frac{3}{y^{3/2}} dy$

$\int 3y^{-3/2} dy = \frac{3y^{-1/2}}{-1/2} = \boxed{\frac{-6}{y^{1/2}} + C}$

7. $\int \frac{3\sqrt{x}(1-x)^2}{\sqrt[3]{x}} dx = \int 3x^{1/6}(1 - 2x + x^2)dx$

$\int \frac{3x^{1/2}(1 - 2x + x^2)}{x^{1/3}}$

$\int 3x^{1/6} - 6x^{7/6} + 3x^{13/6} dx$

$\frac{3x^{7/6}}{7/6} - \frac{6x^{13/6}}{13/6} + \frac{3x^{19/6}}{19/6} + C$

$\boxed{\frac{18}{7}x^{7/6} - \frac{36}{13}x^{13/6} + \frac{18}{19}x^{19/6} + C}$

Key

Calculus 4.1b Notes Trig Integrals and Differential Equations

Review Derivative Trig Rules:

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$

3) $\frac{d}{dx}\cos u = -\sin u \cdot u'$

2) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$

4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$

6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

Integral Trig Rules:

1) $\int \sin u\, du = -\cos u + C$

2) $\int \cos u\, du = \sin u + C$

3) $\int \sec^2 u\, du = \tan u + C$

4) $\int \csc^2 u\, du = -\cot u + C$

5) $\int \sec u \tan u\, du = \sec u + C$

6) $\int \csc u \cot u\, du = -\csc u + C$

Classwork Examples:

1. $\int \frac{\tan x}{\cos x} - \sin x \; dx$

$\int \sec x \tan x - \sin x\, dx$

$\sec x - (-\cos x) + C$

$\boxed{\sec x + \cos x + C}$

2. $\int \frac{\sin x}{\cos^2 x} dx$

$\int \frac{\sin x}{\cos x \cdot \cos x} \cdot dx$

$\int \tan x \sec x\, dx$

$\boxed{\sec x + C}$

3. $\int (1 + \cot^2 x)dx$ $\ast$ think about trig identity $1+\cot^2 x = \csc^2 x$

$\int \csc^2 x\, dx = \boxed{-\cot x + C}$

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x + 4$, $f'(0) = 3$, and $f(1) = 5$.
Find $f(x)$.

* To help distinguish the constants of integration, use "+C" for the first constant and use "+k" for the second constant of integration.

Ex. 7 Given $g''(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

Differential Equations: These are simply equations that involve derivatives.

Steps for solving Differential equations:

1. Rewrite y' as $\frac{dy}{dx}$
2. Separate variables on either side of equation
3. Take the integral of both sides

Solve for C if finding a specific solution/equation to the differential equation

Example 3: Suppose y' = 2. Solve for y.

$\frac{dy}{dx} = 2$

$dy = 2dx$

$\int dy = \int 2dx$

$y = 2x + C$

*we only need to add "+C" once on the right side of equation.

Example 4: Solve this General Differential equation. $\frac{dy}{dx} = x^3$

$\frac{dy}{dx} = x^3$

$dy = x^3 dx$

$\int dy = \int x^3 dx$

$\boxed{y = \frac{x^4}{4} + C}$

Example 5: Solve this Specific differential equation: y' = 3x – 4 and the point (4, 10) is on the graph of y.

$y' = 3x - 4$

$\frac{dy}{dx} = 3x - 4$

$dy = (3x-4)dx$

$\int dy = \int 3x - 4\,dx$

$y = \frac{3x^2}{2} - 4x + C$

plug in (4,10)

$10 = \frac{3}{2}(4)^2 - 4(4) + C$

$10 = 24 - 16 + C$

$10 = 8 + C$

$C = 2$

$\boxed{y = \frac{3}{2}x^2 - 4x + 2}$

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x+4$, $f'(0) = 3$, and $f(1) = 5$. Find $f(x)$.

* To help distinguish the constants of integration, use "+C" for the first constant and use "+K" for the second constant of integration.

$$f''(x) = 6x+4$$

$$f'(x) = \frac{6x^2}{2} + 4x + C$$

use $f'(0)=3$ to solve for C.

$$3 = \frac{6(0)^2}{2} + 4(0) + C$$

$$\underline{\underline{3 = C}} \longrightarrow f'(x) = 3x^2+4x+3$$

$$f'(x) = 3x^2+4x+3$$

$$f(x) = \frac{3x^3}{3} + \frac{4x^2}{2} + 3x + K$$

$$f(x) = x^3 + 2x^2 + 3x + K$$

Use $f(1)=5$ to find K.

$$5 = 1^3 + 2(1)^2 + 3(1) + K$$

$$5 = 6 + K$$

$$\underline{\underline{-1 = K}} \longrightarrow \boxed{f(x) = x^3+2x^2+3x-1}$$

Ex. 7 Given $g''(x) = 12x+6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

$$g'(x) = \frac{12x^2}{2} + 6x + C$$

$$g(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + Cx + K$$

$$g(x) = 2x^3 + 3x^2 + Cx + K$$

$$4 = 2(0)^3 + 3(0)^2 + C(0) + K$$

$$\underline{4 = K}$$

$$g(x) = 2x^3 + 3x^2 + Cx + 4$$

$$-2 = 2(1)^3 + 3(1)^2 + C(1) + 4$$

$$-2 = 2 + 3 + C + 4$$

$$-2 = 9 + C$$

$$\underline{-11 = C}$$

$$\boxed{g(x) = 2x^3+3x^2-11x+4}$$

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x+4$, $f'(0)=3$, and $f(1)=5$. Find $f(x)$.

* To help distinguish the constants of integration, use "+c" for the first constant and use "+k" for the second constant of integration.

$f''(x) = 6x+4$

$f'(x) = \frac{6x^2}{2} + 4x + c$ use $f'(0)=3$ to solve for c.

$3 = \frac{6(0)^2}{2} + 4(0) + c$

$\underline{3 = c}$ $\rightarrow f'(x) = 3x^2+4x+3$

$f'(x) = 3x^2+4x+3$

$f(x) = \frac{3x^3}{3} + \frac{4x^2}{2} + 3x + K$ Use $f(1)=5$ to find K.

$f(x) = x^3+2x^2+3x+K$

$5 = 1^3 + 2(1)^2 + 3(1) + K$

$5 = 6+K$

$\underline{-1 = K}$ $\longrightarrow$ $\boxed{f(x) = x^3+2x^2+3x-1}$

Ex. 7 Given $g''(x) = 12x+6$ and $g(0)=4$ and $g(1)=-2$. Find $g(x)$.

$g'(x) = \frac{12x^2}{2} + 6x + c$

$g(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + cx + K$

$g(x) = 2x^3 + 3x^2 + cx + K$

$4 = 2(0)^3 + 3(0)^2 + c(0) + K$

$\underline{4 = k}$

$g(x) = 2x^3+3x^2+cx+4$

$-2 = 2(1)^3 + 3(1)^2 + c(1) + 4$

$-2 = 2+3+c+4$

$-2 = 9+c$

$\underline{-11 = c}$

$\boxed{g(x) = 2x^3+3x^2-11x+4}$

4.1b p.255-256 #5, 7, 35-41 odd, 47, 48, 55-61 odd

Find the general solution of the differential equation.

5) $\frac{dy}{dt} = 3t^2$ | $\int dy = \int 3t^2\,dt$ | $\boxed{y = t^3 + c}$ | check solution:

$dy = 3t^2\,dt$ | $y = 3\left(\frac{t^3}{3}\right) + c$ | | $\frac{d}{dt}(t^3) = 3t^2$ ✓

7) $\frac{dy}{dx} = x^{3/2}$ | $\int dy = \int x^{3/2}\,dx$ | $\boxed{y = \frac{2}{5}x^{5/2} + c}$ | check: $\frac{d}{dx}\left(\frac{2}{5}x^{5/2} + c\right) = \frac{2}{5}\cdot\frac{5}{2}x^{5/2 - 2/2}$

$dy = x^{3/2}\,dx$ | $y = \frac{x^{5/2}}{5/2} + c$ | | $= x^{3/2}$ ✓

35) $\int (2\sin x + 3\cos x)\,dx$ | $= 2(-\cos x) + 3(\sin x) + c$ | check:

$= 2\int \sin x\,dx + 3\int \cos x\,dx$ | $= \boxed{-2\cos x + 3\sin x + c}$ | $\frac{d}{dx}(-2\cos x + 3\sin x)$

$= -2(-\sin x) + 3\cos x$

$= 2\sin x + 3\cos x$ ✓

37) $\int (1 - \csc t \cot t)\,dt$ | $t - (-\csc t) + c$ | check:

$= \int 1\,dt - \int \csc t \cot t\,dt$ | $= \boxed{t + \csc t + c}$ | $\frac{d}{dt}(t + \csc t)$

$= 1 - \csc t \cot t$ ✓

41) $\int \tan^2 y + 1\,dy$ | $= \boxed{\tan y + c}$ | check:

use trig identity to rewrite problem | | $\frac{d}{dy}(\tan y) = \sec^2 y = 1 + \tan^2 y$ ✓

$\int \sec^2 y\,dy$

4.1b Trig Integrals and Differential Equations

Review:

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$
2) $\frac{d}{dx}\cos u = -\sin u \cdot u'$
3) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$
4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$
5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$
6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

Trig Integrals

1) $\int \sin u\,du = -\cos u + C$
2) $\int \cos u\,du = \sin u + C$
3) $\int \sec^2 u\,du = \tan u + C$
4) $\int \csc^2 u\,du = -\cot u + C$
5) $\int \sec u \tan u\,du = \sec u + C$
6) $\int \csc u \cot u\,du = -\csc u + C$

$\boxed{\text{Ex. 1}}$ $\int (1+\cot^2 x)\,dx$

*Use trig identity to rewrite integrand: $1+\cot^2 x = \csc^2 x$

$$\int \csc^2 x\,dx = -\cot x + C$$

$\boxed{\text{Ex. 2}}$ $\int \frac{\sin x}{\cos^2 x}\,dx$

* Try rewriting in different forms to match integral formula

$$\int \frac{\sin x}{\cos x \cdot \cos x}\,dx$$

$$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$\int \tan x \sec x\,dx$$

$$= \boxed{\sec x + C}$$

4.1b (continued)

Differential Equations: These are simply equations that involves derivatives.

Steps:

1) Rewrite y' as $\frac{dy}{dx}$
2) Separate variables on either side of equation
3) Take Integral of both sides.
4) Solve for C if finding specific differential equation.

Ex. 3 Suppose $y' = 2$. Solve for y.

$$\frac{dy}{dx} = 2$$

$$dy = 2dx$$

$$\int dy = \int 2dx$$

$$y = 2x + C$$

we only have to write "+C" once on right side of equation.

Ex. 4 Solve this General differential equation (like ex. 3)

Solve for y: $\frac{dy}{dx} = x^3$

$$dy = x^3 dx$$

$$\int dy = \int x^3 dx$$

$$\boxed{y = \frac{x^4}{4} + C}$$

Ex. 5 Solve this Specific differential equation

Solve for y: $y' = 3x - 4$ and the point $(4, 10)$ is on graph of y.

$$\frac{dy}{dx} = 3x - 4$$

$$dy = 3x - 4\,dx$$

$$\int dy = \int 3x - 4\,dx$$

$$y = \frac{3x^2}{2} - 4x + C$$ plug in

$$10 = \frac{3(4)^2}{2} - 4(4) + C$$

$$10 = 3(8) - 16 + C$$

$$\underline{\underline{2 = C}}$$

$$\boxed{y = \frac{3}{2}x^2 - 4x + 2}$$

Key

A.P. Calculus AB **Quiz 4-1 Review** **No Calculators**

Find the most general antiderivative of $h(x)$.

1. $h(x) = 5x^4 - 5x^2 + \pi + \frac{1}{2\sqrt{x}} + \frac{1}{3x^3} = 5x^4 - 5x^2 + \pi + \frac{x^{-1/2}}{2} + \frac{1}{3}x^{-3}$

$$H(x) = \int h(x)\,dx = \frac{5x^5}{5} - \frac{5x^3}{3} + \pi x + \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + \frac{1}{3}\left(\frac{x^{-2}}{-2}\right) + C = \boxed{x^5 - \frac{5x^3}{3} + \pi x + \sqrt{x} - \frac{1}{6x^2} + C}$$

2. $h(x) = \frac{-8}{\sqrt[3]{x}} - 2\csc^2 x = -8x^{-1/3} - 2\csc^2 x$

$$\int h(x)\,dx = -8\left(\frac{x^{2/3}}{2/3}\right) - 2(-\cot x)$$
$$= -8\left(\frac{3}{2}\right)\left(x^{2/3}\right) + 2\cot x + C$$
$$= \boxed{-12x^{2/3} + 2\cot x + C}$$

3. $h(x) = \frac{7}{\sqrt[5]{x}} - \frac{\sqrt[4]{x}}{13} + 10 = 7x^{-1/5} - \frac{1}{13}x^{1/4} + 10$

$$\int h(x)\,dx = \frac{7x^{4/5}}{4/5} - \frac{1}{13}\left(\frac{x^{5/4}}{5/4}\right) + 10x + C =$$
$$7\left(\frac{5}{4}\right)x^{4/5} - \frac{1}{13}\left(\frac{4}{5}\right)x^{5/4} + 10x + C$$
$$\boxed{\frac{35}{4}x^{4/5} - \frac{4}{65}x^{5/4} + 10x + C}$$

4. $h(x) = \frac{2x - 6x^3 + x^2}{\sqrt[3]{x}} = \frac{2x^1}{x^{1/3}} - \frac{6x^3}{x^{1/3}} + \frac{x^2}{x^{1/3}} = 2x^{2/3} - 6x^{8/3} + x^{5/3}$

$$\int h(x)\,dx = 2\left(\frac{x^{5/3}}{5/3}\right) - 6\left(\frac{x^{11/3}}{11/3}\right) + \frac{x^{8/3}}{8/3} + C$$
$$= 2\left(\frac{3}{5}\right)x^{5/3} - 6\left(\frac{3}{11}\right)x^{11/3} + \frac{3}{8}x^{8/3} + C$$
$$\boxed{\frac{6x^{5/3}}{5} - \frac{18x^{11/3}}{11} + \frac{3x^{8/3}}{8} + C}$$

5. $h(x) = -2\cos x + 5\sin x - 5\csc x\cot x$

$$\int h(x)\,dx = -2(\sin x) + 5(-\cos x) - 5(-\csc x) + C$$
$$= \boxed{-2\sin x - 5\cos x + 5\csc x + C}$$

6. Find the most general expression of $f(x)$ if $f''(x) = 9x^2 - 5x + 2$.

$$f'(x) = \frac{9x^3}{3} - \frac{5x^2}{2} + 2x + c = 3x^3 - \frac{5x^2}{2} + 2x + c$$

$$f(x) = \frac{3x^4}{4} - \frac{5}{2}\left(\frac{x^3}{3}\right) + \frac{2x^2}{2} + c_1x + C_2$$

$$= \boxed{\frac{3}{4}x^4 - \frac{5x^3}{6} + x^2 + c_1x + C_2}$$

7. Find the most general expression of $f(x)$ if $f''(x) = 4x^3 - 5x^2 + 3x - 6$.

$$f'(x) = \frac{4x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + c$$

$$= x^4 - \frac{5x^3}{3} + \frac{3x^2}{2} - 6x + c$$

$$f(x) = \frac{x^5}{5} - \frac{5}{3}\left(\frac{x^4}{4}\right) + \frac{3}{2}\left(\frac{x^3}{3}\right) - \frac{6x^2}{2} + c_1x + C_2$$

$$\boxed{f(x) = \frac{x^5}{5} - \frac{5x^4}{12} + \frac{x^3}{2} - 3x^2 + c_1x + C_2}$$

8. Find the specific expression of $f(x)$ if $f(x) = \int g(x)dx$, $g(x) = 3x^2 - 4x$, and $f(-1) = 2$

$$\int g(x)dx = \int 3x^2 - 4x\,dx = \frac{3x^3}{3} - \frac{4x^2}{2} + c \rightarrow f(x) = x^3 - 2x^2 + c$$

$$f(-1) = (-1)^3 - 2(-1)^2 + c$$

$$2 = -1 - 2 + c$$

$$2 + 3 = c$$

$$5 = c$$

$$\boxed{f(x) = x^3 - 2x^2 + 5}$$

9. Find the specific expression of $f(x)$ if $f''(x) = 36x^2 - 6$, $f'(-1) = 3$, and $f(1) = 9$.

$$f'(x) = \frac{36x^3}{3} - 6x + c = 12x^3 - 6x + c_1$$

$$f'(-1) = 12(-1)^3 - 6(-1) + c_1$$

$$3 = -12 + 6 + c_1$$

$$3 = -6 + c_1$$

$$9 = c_1$$

$$f'(x) = 12x^3 - 6x + 9$$

$$f(x) = \frac{12x^4}{4} - \frac{6x^2}{2} + 9x + C_2$$

$$f(x) = 3x^4 - 3x^2 + 9x + C_2$$

$$f(1) = 3(1)^4 - 3(1)^2 + 9(1) + C_2$$

$$9 = 3 - 3 + 9 + C_2$$

$$9 = 0 + 9 + C_2$$

$$0 = C_2$$

$$\boxed{f(x) = 3x^4 - 3x^2 + 9x}$$