HAPTER 4
ntegration

ction 4.1 Antiderivatives and Indefinite Integration

1. $\frac{d}{dx}\left(\frac{2}{x^3} + C\right) = \frac{d}{dx}\left(2x^{-3} + C\right) = -6x^{-4} = \frac{-6}{x^4}$

2. $\frac{d}{dx}\left(2x^4 - \frac{1}{2x} + C\right) = \frac{d}{dx}\left(2x^4 - \frac{1}{2}x^{-1} + C\right)$

$= 8x^3 + \frac{1}{2}x^{-2} = 8x^3 + \frac{1}{2x^2}$

3. $\frac{dy}{dt} = 9t^2$

$y = 3t^3 + C$

Check: $\frac{d}{dt}\left[3t^3 + C\right] = 9t^2$

4. $\frac{dy}{dt} = 5$

$y = 5t + C$

Check: $\frac{d}{dt}[5t + C] = 5$

5. $\frac{dy}{dx} = x^{3/2}$

$y = \frac{2}{5}x^{5/2} + C$

Check: $\frac{d}{dx}\left[\frac{2}{5}x^{5/2} + C\right] = x^{3/2}$

6. $\frac{dy}{dx} = 2x^{-3}$

$y = \frac{2x^{-2}}{-2} + C = \frac{-1}{x^2} + C$

Check: $\frac{d}{dx}\left[\frac{-1}{x^2} + C\right] = 2x^{-3}$

	Given	*Rewrite*	*Integrate*	*Simplify*
7.	$\int \sqrt[3]{x}\, dx$	$\int x^{1/3}\, dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
8.	$\int \frac{1}{4x^2}\, dx$	$\frac{1}{4}\int x^{-2}\, dx$	$\frac{1}{4}\frac{x^{-1}}{-1} + C$	$-\frac{1}{4x} + C$
9.	$\int \frac{1}{x\sqrt{x}}\, dx$	$\int x^{-3/2}\, dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$
10.	$\int \frac{1}{(3x)^2}\, dx$	$\frac{1}{9}\int x^{-2}\, dx$	$\frac{1}{9}\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{-1}{9x} + C$

11. $\int (x + 7)\, dx = \frac{x^2}{2} + 7x + C$

Check: $\frac{d}{dx}\left[\frac{x^2}{2} + 7x + C\right] = x + 7$

12. $\int (13 - x)\, dx = 13x - \frac{x^2}{2} + C$

Check: $\frac{d}{dx}\left[13x - \frac{x^2}{2} + C\right] = 13 - x$

13. $\int \left(x^5 + 1\right) dx = \frac{x^6}{6} + x + C$

Check: $\frac{d}{dx}\left(\frac{x^6}{6} + x + C\right) = x^5 + 1$

14. $\int(8x^3 - 9x^2 + 4)\,dx = 2x^4 - 3x^3 + 4x + C$

Check: $\frac{d}{dx}(2x^4 - 3x^3 + 4x + C) = 8x^3 - 9x^2 + 4$

15. $\int(x^{3/2} + 2x + 1)\,dx = \frac{2}{5}x^{5/2} + x^2 + x + C$

Check: $\frac{d}{dx}\left(\frac{2}{5}x^{5/2} + x^2 + x + C\right) = x^{3/2} + 2x + 1$

16. $\int\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx = \int\left(x^{1/2} + \frac{1}{2}x^{-1/2}\right)dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + C$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

Check: $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + x^{1/2} + C\right) = x^{1/2} + \frac{1}{2}x^{-1/2}$

$= \sqrt{x} + \frac{1}{2\sqrt{x}}$

17. $\int\sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx = \frac{x^{5/3}}{5/3} + C = \frac{3}{5}x^{5/3} + C$

Check: $\frac{d}{dx}\left(\frac{3}{5}x^{5/3} + C\right) = x^{2/3} = \sqrt[3]{x^2}$

18. $\int\left(\sqrt[4]{x^3} + 1\right)dx = \int\left(x^{3/4} + 1\right)dx = \frac{4}{7}x^{7/4} + x + C$

Check: $\frac{d}{dx}\left(\frac{4}{7}x^{7/4} + x + C\right) = x^{3/4} + 1 = \sqrt[4]{x^3} + 1$

19. $\int\frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-4}}{-4} + C = \frac{-1}{4x^4} + C$

Check: $\frac{d}{dx}\left(\frac{-1}{4x^4} + C\right) = \frac{d}{dx}\left(-\frac{1}{4}x^{-4} + C\right)$

$= -\frac{1}{4}\left(-4x^{-5}\right) = \frac{1}{x^5}$

20. $\int\frac{3}{x^7}\,dx = \int 3x^{-7}\,dx = \frac{3x^{-6}}{-6} + C = -\frac{1}{2x^6} + C$

Check: $\frac{d}{dx}\left(-\frac{1}{2x^6} + C\right) = \frac{d}{dx}\left(-\frac{1}{2}x^{-6} + C\right)$

$= \left(-\frac{1}{2}\right)(-6)x^{-7} = \frac{3}{x^7}$

21. $\int\frac{x + 6}{\sqrt{x}}\,dx = \int\left(x^{1/2} + 6x^{-1/2}\right)dx$

$= \frac{x^{3/2}}{3/2} + 6\frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3}x^{3/2} + 12x^{1/2} + C$

$= \frac{2}{3}x^{1/2}(x + 18) + C$

Check: $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + 12x^{1/2} + C\right)$

$= \frac{2}{3}\left(\frac{3}{2}x^{1/2}\right) + 12\left(\frac{1}{2}x^{-1/2}\right)$

$= x^{1/2} + 6x^{-1/2} = \frac{x + 6}{\sqrt{x}}$

22. $\int\frac{x^4 - 3x^2 + 5}{x^4}\,dx = \int\left(1 - 3x^{-2} + 5x^{-4}\right)dx$

$= x - \frac{3x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C$

$= x + \frac{3}{x} - \frac{5}{3x^3} + C$

Check:

$\frac{d}{dx}\left[x + \frac{3}{x} - \frac{5}{3x^3} + C\right] = \frac{d}{dx}\left[x + 3x^{-1} - \frac{5}{3}x^{-3} + C\right]$

$= 1 - 3x^{-2} + 5x^{-4}$

$= 1 - \frac{3}{x^2} + \frac{5}{x^4}$

$= \frac{x^4 - 3x^2 + 5}{x^4}$

23. $\int(x + 1)(3x - 2)\,dx = \int\left(3x^2 + x - 2\right)dx$

$= x^3 + \frac{1}{2}x^2 - 2x + C$

Check: $\frac{d}{dx}\left(x^3 + \frac{1}{2}x^2 - 2x + C\right) = 3x^2 + x - 2$

$= (x + 1)(3x - 2)$

24. $\int\left(4t^2 + 3\right)^2 dt = \int\left(16t^4 + 24t^2 + 9\right)dt$

$= \frac{16t^5}{5} + 8t^3 + 9t + C$

Check: $\frac{d}{dt}\left(\frac{16t^5}{5} + 8t^3 + 9t + C\right) = 16t^4 + 24t^2 + 9$

$= \left(4t^2 + 3\right)^2$

25. $\int (5 \cos x + 4 \sin x)\, dx = 5 \sin x - 4 \cos x + C$

Check: $\frac{d}{dx}(5 \sin x - 4 \cos x + C) = 5 \cos x + 4 \sin x$

26. $\int (t^2 - \cos t)\, dt = \frac{t^3}{3} - \sin t + C$

Check: $\frac{d}{dt}\left(\frac{t^3}{3} - \sin t + C\right) = t^2 - \cos t$

27. $\int (1 - \csc t \cot t)\, dt = t + \csc t + C$

Check: $\frac{d}{dt}(t + \csc t + C) = 1 - \csc t \cot t$

28. $\int (\theta^2 + \sec^2 \theta)\, d\theta = \frac{1}{3}\theta^3 + \tan \theta + C$

Check: $\frac{d}{d\theta}\left(\frac{1}{3}\theta^3 + \tan \theta + C\right) = \theta^2 + \sec^2 \theta$

29. $\int (\sec^2 \theta - \sin \theta)\, d\theta = \tan \theta + \cos \theta + C$

Check: $\frac{d}{d\theta}(\tan \theta + \cos \theta + C) = \sec^2 \theta - \sin \theta$

30. $\int \sec y(\tan y - \sec y)\, dy = \int (\sec y \tan y - \sec^2 y)\, dy$

$= \sec y - \tan y + C$

Check: $\frac{d}{dy}(\sec y - \tan y + C) = \sec y \tan y - \sec^2 y$

$= \sec y(\tan y - \sec y)$

31. $\int (\tan^2 y + 1)\, dy = \int \sec^2 y\, dy = \tan y + C$

Check: $\frac{d}{dy}(\tan y + C) = \sec^2 y = \tan^2 y + 1$

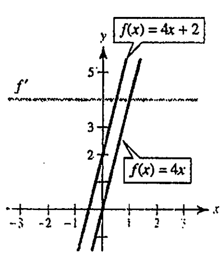
32. $\int (4x - \csc^2 x)\, dx = 2x^2 + \cot x + C$

Check: $\frac{d}{dx}(2x^2 + \cot x + C) = 4x - \csc^2 x$

33. $f'(x) = 4$

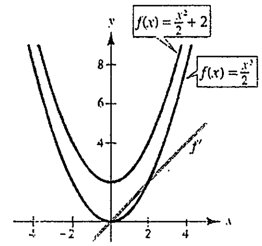
$f(x) = 4x + C$

Answers will vary.



34. $f'(x) = x$

$f(x) = \frac{x^2}{2} + C$



Answers will vary.

35. $f'(x) = 6x,\ f(0) = 8$

$f(x) = \int 6x\, dx = 3x^2 + C$

$f(0) = 8 = 3(0)^2 + C \Rightarrow C = 8$

$f(x) = 3x^2 + 8$

36. $g'(x) = 4x^2,\ g(-1) = 3$

$g(x) = \int 4x^2\, dx = \frac{4}{3}x^3 + C$

$g(-1) = 3 = -\frac{4}{3} = C \Rightarrow C = \frac{13}{3}$

$g(x) = \frac{4}{3}x^3 + \frac{13}{3}$

37. $h'(t) = 8t^3 + 5,\ h(1) = -4$

$h(t) = \int (8t^3 + 5)\, dt = 2t^4 + 5t + C$

$h(1) = -4 = 2 + 5 + C \Rightarrow C = -11$

$h(t) = 2t^4 + 5t - 11$

38. $f'(s) = 10s - 12s^3, f(3) = 2$

$f(s) = \int (10s - 12s^3)\, ds = 5s^2 - 3s^4 + C$

$f(3) = 2 = 5(3)^2 - 3(3)^4 + C = 45 - 243 + C \Rightarrow C = 200$

$f(s) = 5s^2 - 3s^4 + 200$

39. $f''(x) = 2$

$f'(2) = 5$

$f(2) = 10$

$f'(x) = \int 2\, dx = 2x + C_1$

$f'(2) = 4 + C_1 = 5 \Rightarrow C_1 = 1$

$f'(x) = 2x + 1$

$f(x) = \int (2x + 1)\, dx = x^2 + x + C_2$

$f(2) = 6 + C_2 = 10 \Rightarrow C_2 = 4$

$f(x) = x^2 + x + 4$

40. $f''(x) = x^2$

$f'(0) = 8$

$f(0) = 4$

$f'(x) = \int x^2\,dx = \frac{1}{3}x^3 + C_1$

$f'(0) = 0 + C_1 = 8 \Rightarrow C_1 = 8$

$f'(x) = \frac{1}{3}x^3 + 8$

$f(x) = \int\left(\frac{1}{3}x^3 + 8\right)dx = \frac{1}{12}x^4 + 8x + C_2$

$f(0) = 0 + 0 + C_2 = 4 \Rightarrow C_2 = 4$

$f(x) = \frac{1}{12}x^4 + 8x + 4$

41. $f''(x) = x^{-3/2}$

$f'(4) = 2$

$f(0) = 0$

$f'(x) = \int x^{-3/2}\,dx = -2x^{-1/2} + C_1 = -\frac{2}{\sqrt{x}} + C_1$

$f'(4) = -\frac{2}{2} + C_1 = 2 \Rightarrow C_1 = 3$

$f'(x) = -\frac{2}{\sqrt{x}} + 3$

$f(x) = \int\left(-2x^{-1/2} + 3\right)dx = -4x^{1/2} + 3x + C_2$

$f(0) = 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$

$f(x) = -4x^{1/2} + 3x = -4\sqrt{x} + 3x$

42. $f''(x) = \sin x$

$f'(0) = 1$

$f(0) = 6$

$f'(x) = \int \sin x\,dx = -\cos x + C_1$

$f'(0) = -1 + C_1 = 1 \Rightarrow C_1 = 2$

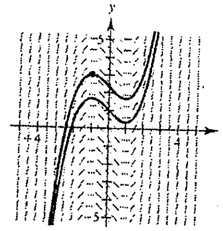
$f'(x) = -\cos x + 2$

$f(x) = \int(-\cos x + 2)\,dx = -\sin x + 2x + C_2$

$f(0) = 0 + 0 + C_2 = 6 \Rightarrow C_2 = 6$

$f(x) = -\sin x + 2x + 6$

43. (a) Answers will vary. *Sample answer:*



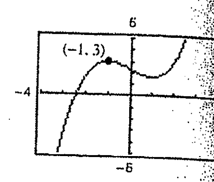
(b) $\frac{dy}{dx} = x^2 - 1, (-1, 3)$

$y = \frac{x^3}{3} - x + C$

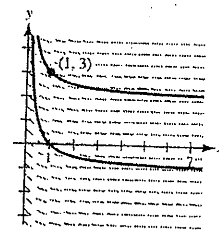
$3 = \frac{(-1)^3}{3} - (-1) + C$

$C = \frac{7}{3}$

$y = \frac{x^3}{3} - x + \frac{7}{3}$



44. (a) Answers will vary. *Sample answer:*

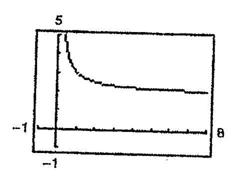


(b) $\frac{dy}{dx} = \frac{-1}{x^2}, x > 0, (1, 3)$

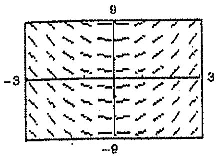
$y = \int -\frac{1}{x^2}\,dx = \int -x^{-2}\,dx = \frac{-x^{-1}}{-1} + C = \frac{1}{x} + C$

$3 = \frac{1}{1} + C \Rightarrow C = 2$

$y = \frac{1}{x} + 2$



45. (a)



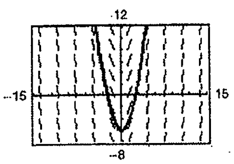
(b) $\dfrac{dy}{dx} = 2x, (-2, -2)$

$y = \int 2x\,dx = x^2 + C$

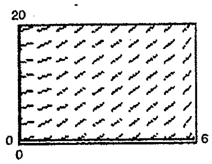
$-2 = (-2)^2 + C = 4 + C \Rightarrow C = -6$

$y = x^2 - 6$

(c)



46. (a)



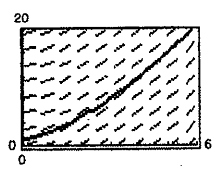
(b) $\dfrac{dy}{dx} = 2\sqrt{x}, (4, 12)$

$y = \int 2x^{1/2}\,dx = \dfrac{4}{3}x^{3/2} + C$

$12 = \dfrac{4}{3}(4)^{3/2} + C = \dfrac{4}{3}(8) + C = \dfrac{32}{3} + C \Rightarrow C = \dfrac{4}{3}$

$y = \dfrac{4}{3}x^{3/2} + \dfrac{4}{3}$

(c)



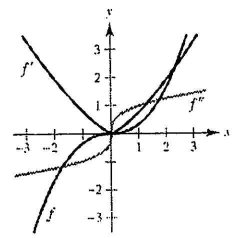
47. They are the same. In both cases you are finding a function $F(x)$ such that $F'(x) = f(x)$.

48. $f(x) = \tan^2 x \Rightarrow f'(x) = 2\tan x \cdot \sec^2 x$

$g(x) = \sec^2 x \Rightarrow g'(x) = 2\sec x \cdot \sec x \tan x = f'(x)$

The derivatives are the same, so f and g differ by a constant. In fact, $\tan^2 x + 1 = \sec^2 x$.

49. Because f'' is negative on $(-\infty, 0)$, f' is decreasing on $(-\infty, 0)$. Because f'' is positive on $(0, \infty)$, f' is increasing on $(0, \infty)$. f' has a relative minimum at $(0, 0)$. Because f' is positive on $(-\infty, \infty)$, f is increasing on $(-\infty, \infty)$.



50. $f(0) = -4$. Graph of f' is given.

(a) $f'(4) \approx -1.0$

(b) No. The slopes of the tangent lines are greater than 2 on $[0, 2]$. Therefore, f must increase more than 4 units on $[0, 4]$.

(c) No, $f(5) < f(4)$ because f is decreasing on $[4, 5]$.

(d) f is a maximum at $x = 3.5$ because $f'(3.5) \approx 0$ and the First Derivative Test.

(e) f is concave upward when f' is increasing on $(-\infty, 1)$ and $(5, \infty)$. f is concave downward on $(1, 5)$. Points of inflection at $x = 1, 5$.

(f) f'' is a minimum at $x = 3$.

(g) NEED NEW ART

51. (a) $h(t) = \int (1.5t + 5)\,dt = 0.75t^2 + 5t + C$

$h(0) = 0 + 0 + C = 12 \Rightarrow C = 12$

$h(t) = 0.75t^2 + 5t + 12$

(b) $h(6) = 0.75(6)^2 + 5(6) + 12 = 69$ cm

52. $\frac{dP}{dt} = k\sqrt{t},\ 0 \le t \le 10$

$P(t) = \int kt^{1/2}\,dt = \frac{2}{3}kt^{3/2} + C$

$P(0) = 0 + C = 500 \Rightarrow C = 500$

$P(1) = \frac{2}{3}k + 500 = 600 \Rightarrow k = 150$

$P(t) = \frac{2}{3}(150)t^{3/2} + 500 = 100t^{3/2} + 500$

$P(7) = 100(7)^{3/2} + 500 \approx 2352$ bacteria

53. $a(t) = -32\text{ ft/sec}^2$

$v(t) = \int -32\,dt = -32t + C_1$

$v(0) = 60 = C_1$

$s(t) = \int(-32t + 60)\,dt = -16t^2 + 60t + C_2$

$s(0) = 6 = C_2$

$s(t) = -16t^2 + 60t + 6$, Position function

The ball reaches its maximum height when

$v(t) = -32t + 60 = 0$

$32t = 60$

$t = \frac{15}{8}$ seconds.

$s\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) + 6 = 62.25$ feet

54. $a(t) = -32\text{ ft/sec}^2$

$v(t) = \int -32\,dt = -32t + C_1$

$v(0) = 0 + C_1 = V_0 \Rightarrow C_1 = V_0$

$s'(t) = -32t + V_0$

$s(t) = \int(-32t + V_0)dt = -16t^2 + V_0t + C_2$

$s(0) = 0 + 0 + C_2 = S_0 \Rightarrow C_2 = S_0$

$s(t) = -16t^2 + V_0t + S_0$

$s'(t) = -32t + v_0 = 0$ when $t = \frac{v_0}{32}$ = time to reach maximum height.

$$s\left(\frac{v_0}{32}\right) = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) = 550$$

$$-\frac{v_0^2}{64} + \frac{v_0^2}{32} = 550$$

$$v_0^2 = 35{,}200$$

$$v_0 \approx 187.617\text{ ft/sec}$$

55. $v_0 = 16$ ft/sec

$s_0 = 64$ ft

(a) $s(t) = -16t^2 + 16t + 64 = 0$

$-16(t^2 - t - 4) = 0$

$t = \frac{1 \pm \sqrt{17}}{2}$

Choosing the positive value,

$t = \frac{1 + \sqrt{17}}{2} \approx 2.562$ seconds.

(b) $v(t) = s'(t) = -32t + 16$

$v\left(\frac{1 + \sqrt{17}}{2}\right) = -32\left(\frac{1 + \sqrt{17}}{2}\right) + 16$

$= -16\sqrt{17} \approx -65.970$ ft/sec

56. $a(t) = -9.8$

$v(t) = \int -9.8\,dt = -9.8t + C_1$

$v(0) = v_0 = C_1 \Rightarrow v(t) = -9.8t + v_0$

$f(t) = \int(-9.8t + v_0)\,dt = -4.9t^2 + v_0t + C_2$

$f(0) = s_0 = C_2 \Rightarrow f(t) = -4.9t^2 + v_0t + s_0$

So, $f(t) = -4.9t^2 + 10t + 2$.

$v(t) = -9.8t + 10 = 0$ (Maximum height when $v = 0$.)

$9.8t = 10$

$t = \frac{10}{9.8}$

$f\left(\frac{10}{9.8}\right) \approx 7.1$ m

57. From Exercise 56, $f(t) = -4.9t^2 + v_0t + 2$. If

$f(t) = 200 = -4.9t^2 + v_0t + 2,$

then

$v(t) = -9.8t + v_0 = 0$

for this t value. So, $t = v_0/9.8$ and you solve

$$-4.9\left(\frac{v_0}{9.8}\right)^2 + v_0\left(\frac{v_0}{9.8}\right) + 2 = 200$$

$$\frac{-4.9v_0^2}{(9.8)^2} + \left(\frac{v_0^2}{9.8}\right) = 198$$

$$-4.9v_0^2 + 9.8v_0^2 = (9.8)^2 198$$

$$4.9v_0^2 = (9.8)^2 198$$

$$v_0^2 = 3880.8$$

$$v_0 \approx 62.3\text{ m/sec.}$$

58. From Exercise 56, $f(t) = -4.9t^2 + 1800$. (Using the canyon floor as position 0.)

$$f(t) = 0 = -4.9t^2 + 1800$$
$$4.9t^2 = 1800$$
$$t^2 = \frac{1800}{4.9} \Rightarrow t \approx 9.2 \text{ sec}$$

59. $a = -1.6$

$v(t) = \int -1.6\, dt = -1.6t + v_0 = -1.6t$, because the stone was dropped, $v_0 = 0$.

$$s(t) = \int (-1.6t)\, dt = -0.8t^2 + s_0$$
$$s(20) = 0 \Rightarrow -0.8(20)^2 + s_0 = 0$$
$$s_0 = 320$$

So, the height of the cliff is 320 meters.

$$v(t) = -1.6t$$
$$v(20) = -32 \text{ m/sec}$$

60. $\int v\, dv = -GM \int \frac{1}{y^2}\, dy$

$$\frac{1}{2}v^2 = \frac{GM}{y} + C$$

When $y = R$, $v = v_0$.

$$\frac{1}{2}v_0^2 = \frac{GM}{R} + C$$
$$C = \frac{1}{2}v_0^2 - \frac{GM}{R}$$
$$\frac{1}{2}v^2 = \frac{GM}{y} + \frac{1}{2}v_0^2 - \frac{GM}{R}$$
$$v^2 = \frac{2GM}{y} + v_0^2 - \frac{2GM}{R}$$
$$v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$$

61. $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) $v(t) = x'(t) = 3t^2 - 12t + 9$
$= 3(t^2 - 4t + 3) = 3(t - 1)(t - 3)$
$a(t) = v'(t) = 6t - 12 = 6(t - 2)$

(b) $v(t) > 0$ when $0 < t < 1$ or $3 < t < 5$.

(c) $a(t) = 6(t - 2) = 0$ when $t = 2$.
$v(2) = 3(1)(-1) = -3$

62. $x(t) = (t - 1)(t - 3)^2 \quad 0 \le t \le 5$
$= t^3 - 7t^2 + 15t - 9$

(a) $v(t) = x'(t) = 3t^2 - 14t + 15 = (3t - 5)(t - 3)$
$a(t) = v'(t) = 6t - 14$

(b) $v(t) > 0$ when $0 < t < \frac{5}{3}$ and $3 < t < 5$.

(c) $a(t) = 6t - 14 = 0$ when $t = \frac{7}{3}$.
$v\left(\frac{7}{3}\right) = \left(3\left(\frac{7}{3}\right) - 5\right)\left(\frac{7}{3} - 3\right) = 2\left(-\frac{2}{3}\right) = -\frac{4}{3}$

63. $v(t) = \frac{1}{\sqrt{t}} = t^{-1/2} \quad t > 0$

$x(t) = \int v(t)dt = 2t^{1/2} + C$

$x(1) = 4 = 2(1) + C \Rightarrow C = 2$

Position function: $x(t) = 2t^{1/2} + 2$

Acceleration function: $a(t) = v'(t) = -\frac{1}{2}t^{-3/2} = \frac{-1}{2t^{3/2}}$

64. (a) $a(t) = \cos t$
$v(t) = \int a(t)\, dt$
$= \int \cos t\, dt$
$= \sin t + C_1 = \sin t$ (because $v_0 = 0$)
$f(t) = \int v(t)\, dt = \int \sin t\, dt = -\cos t + C_2$
$f(0) = 3 = -\cos(0) + C_2 = -1 + C_2 \Rightarrow C_2 = 4$
$f(t) = -\cos t + 4$

(b) $v(t) = 0 = \sin t$ for $t = k\pi$, $k = 0, 1, 2, \ldots$

65. (a) $v(0) = 25 \text{ km/h} = 25 \cdot \frac{1000}{3600} = \frac{250}{36}$ m/sec
$v(13) = 80 \text{ km/h} = 80 \cdot \frac{1000}{3600} = \frac{800}{36}$ m/sec
$a(t) = a$ (constant acceleration)
$v(t) = at + C$
$v(0) = \frac{250}{36} \Rightarrow v(t) = at + \frac{250}{36}$
$v(13) = \frac{800}{36} = 13a + \frac{250}{36}$
$\frac{550}{36} = 13a$
$a = \frac{550}{468} = \frac{275}{234} \approx 1.175 \text{ m/sec}^2$

(b) $s(t) = a\frac{t^2}{2} + \frac{250}{36}t \quad (s(0) = 0)$

$s(13) = \frac{275}{234}\frac{(13)^2}{2} + \frac{250}{36}(13) \approx 189.58$ m

66. $v(0) = 45 \text{ mi/h} = 66 \text{ ft/sec}$

$30 \text{ mi/h} = 44 \text{ ft/sec}$

$15 \text{ mi/h} = 22 \text{ ft/sec}$

$a(t) = -a$

$v(t) = -at + 66$

$s(t) = -\frac{a}{2}t^2 + 66t$ (Let $s(0) = 0$.)

$v(t) = 0$ after car moves 132 ft.

$-at + 66 = 0$ when $t = \frac{66}{a}$.

$$s\left(\frac{66}{a}\right) = -\frac{a}{2}\left(\frac{66}{a}\right)^2 + 66\left(\frac{66}{a}\right)$$

$$= 132 \text{ when } a = \frac{33}{2} = 16.5.$$

$a(t) = -16.5$

$v(t) = -16.5t + 66$

$s(t) = -8.25t^2 + 66t$

(a) $-16.5t + 66 = 44$

$$t = \frac{22}{16.5} \approx 1.333$$

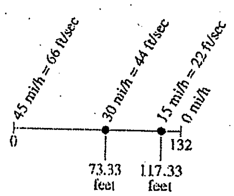
$$s\left(\frac{22}{16.5}\right) \approx 73.33 \text{ ft}$$

(b) $-16.5t + 66 = 22$

$$t = \frac{44}{16.5} \approx 2.667$$

$$s\left(\frac{44}{16.5}\right) \approx 117.33 \text{ ft}$$

(c)



It takes 1.333 seconds to reduce the speed from 45 mi/h to 30 mi/h, 1.333 seconds to reduce the speed from 30 mi/h to 15 mi/h, and 1.333 seconds to reduce the speed from 15 mi/h to 0 mi/h. Each time, less distance is needed to reach the next speed reduction.

67. Truck: $v(t) = 30$

$s(t) = 30t$ (Let $s(0) = 0$.)

Automobile: $a(t) = 6$

$v(t) = 6t$ (Let $v(0) = 0$.)

$s(t) = 3t^2$ (Let $s(0) = 0$.)

At the point where the automobile overtakes the truck:

$30t = 3t^2$

$0 = 3t^2 - 30t$

$0 = 3t(t - 10)$ when $t = 10$ sec.

(a) $s(10) = 3(10)^2 = 300$ ft

(b) $v(10) = 6(10) = 60 \text{ ft/sec} \approx 41 \text{ mi/h}$

68. $a(t) = k$

$v(t) = kt$

$s(t) = \frac{k}{2}t^2$ because $v(0) = s(0) = 0$.

At the time of lift-off, $kt = 160$ and $(k/2)t^2 = 0.7$. Because $(k/2)t^2 = 0.7$,

$$t = \sqrt{\frac{1.4}{k}}$$

$$v\left(\sqrt{\frac{1.4}{k}}\right) = k\sqrt{\frac{1.4}{k}} = 160$$

$$1.4k = 160^2 \Rightarrow k = \frac{160^2}{1.4}$$

$$\approx 18{,}285.714 \text{ mi/h}^2$$

$$\approx 7.45 \text{ ft/sec}^2.$$

69. False. f has an infinite number of antiderivatives, each differing by a constant.

70. True

71. True

72. True

73. True

74. False. For example,

$\int x \cdot x\, dx \neq \int x\, dx \cdot \int x\, dx$ because

$$\frac{x^3}{3} + C \neq \left(\frac{x^2}{2} + C_1\right)\left(\frac{x^2}{2} + C_2\right).$$

75. $f''(x) = 2x$

$f'(x) = x^2 + C$

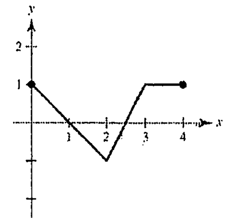
$f'(2) = 0 \Rightarrow 4 + C = 0 \Rightarrow C = -4$

$f(x) = \dfrac{x^3}{3} - 4x + C_1$

$f(2) = 0 \Rightarrow \dfrac{8}{3} - 8 + C_1 = 0 \Rightarrow C_1 = \dfrac{16}{3}$

$f(x) = \dfrac{x^3}{3} - 4x + \dfrac{16}{3}$

76. $f'(x) = \begin{cases} -1, & 0 \le x < 2 \\ 2, & 2 < x < 3 \\ 0, & 3 < x \le 4 \end{cases}$



$f(x) = \begin{cases} -x + C_1, & 0 \le x < 2 \\ 2x + C_2, & 2 < x < 3 \\ C_3, & 3 < x \le 4 \end{cases}$

$f(0) = 1 \Rightarrow C_1 = 1$

f continuous at
$x = 2 \Rightarrow -2 + 1 = 4 + C_2 \Rightarrow C_2 = -5$

f continuous at $x = 3 \Rightarrow 6 - 5 = C_3 = 1$

$f(x) = \begin{cases} -x + 1, & 0 \le x < 2 \\ 2x - 5, & 2 \le x < 3 \\ 1, & 3 \le x \le 4 \end{cases}$

77. $\dfrac{d}{dx}\left[\left[s(x)\right]^2 + \left[c(x)\right]^2\right] = 2s(x)s'(x) + 2c(x)c'(x)$

$= 2s(x)c(x) - 2c(x)s(x) = 0$

So, $\left[s(x)\right]^2 + \left[c(x)\right]^2 = k$ for some constant k.
Because, $s(0) = 0$ and $c(0) = 1$, $k = 1$.

Therefore, $\left[s(x)\right]^2 + \left[c(x)\right]^2 = 1$.

[Note that $s(x) = \sin x$ and $c(x) = \cos x$ satisfy these properties.]

78. $f(x + y) = f(x)f(y) - g(x)g(y)$

$g(x + y) = f(x)g(y) + g(x)f(y)$

$f'(0) = 0$

[Note: $f(x) = \cos x$ and $g(x) = \sin x$ satisfy these conditions]

$f'(x + y) = f(x)f'(y) - g(x)g'(y)$ (Differentiate with respect to y)

$g'(x + y) = f(x)g'(y) + g(x)f'(y)$ (Differentiate with respect to y)

Letting $y = 0$, $f'(x) = f(x)f'(0) - g(x)g'(0) = -g(x)g'(0)$

$g'(x) = f(x)g'(0) + g(x)f'(0) = f(x)g'(0)$

So, $2f(x)f'(x) = -2f(x)g(x)g'(0)$

$2g(x)g'(x) = 2g(x)f(x)g'(0)$.

Adding, $2f(x)f'(x) + 2g(x)g'(x) = 0$.

Integrating, $f(x)^2 + g(x)^2 = C$.

Clearly $C \ne 0$, for if $C = 0$, then $f(x)^2 = -g(x)^2 \Rightarrow f(x) = g(x) = 0$, which contradicts that f, g are nonconstant.

Now, $C = f(x + y)^2 + g(x + y)^2 = \left(f(x)f(y) - g(x)g(y)\right)^2 + \left(f(x)g(y) + g(x)f(y)\right)^2$

$= f(x)^2 f(y)^2 + g(x)^2 g(y)^2 + f(x)^2 g(y)^2 + g(x)^2 f(y)^2$

$= \left[f(x)^2 + g(x)^2\right]\left[f(y)^2 + g(y)^2\right] = C^2$

So, $C = 1$ and you have $f(x)^2 + g(x)^2 = 1$.

Section 4.2 Area

1. $\sum_{i=1}^{6}(3i + 2) = 3\sum_{i=1}^{6} i + \sum_{i=1}^{6} 2 = 3(1 + 2 + 3 + 4 + 5 + 6) + 12 = 75$

2. $\sum_{k=3}^{9}(k^2 + 1) = (3^2 + 1) + (4^2 + 1) + \ldots + (9^2 + 1) = 287$

3. $\sum_{k=0}^{4}\frac{1}{k^2 + 1} = 1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} = \frac{158}{85}$

4. $\sum_{j=4}^{6}\frac{3}{j} = \frac{3}{4} + \frac{3}{5} + \frac{3}{6} = \frac{37}{20}$

5. $\sum_{k=1}^{4} c = c + c + c + c = 4c$

6. $\sum_{i=1}^{4}\left[(i - 1)^2 + (i + 1)^3\right] = (0 + 8) + (1 + 27) + (4 + 64) + (9 + 125) = 238$

7. $\sum_{i=1}^{11}\frac{1}{5i}$

8. $\sum_{i=1}^{14}\frac{9}{1 + i}$

9. $\sum_{j=1}^{6}\left[7\left(\frac{j}{6}\right) + 5\right]$

10. $\sum_{j=1}^{4}\left[1 - \left(\frac{j}{4}\right)^2\right]$

11. $\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3 - \left(\frac{2i}{n}\right)\right]$

12. $\frac{3}{n}\sum_{i=1}^{n}\left[2\left(1 + \frac{3i}{n}\right)^2\right]$

13. $\sum_{i=1}^{12} 7 = 7(12) = 84$

14. $\sum_{i=1}^{30}(-18) = (-18)(30) = -540$

15. $\sum_{i=1}^{24} 4i = 4\sum_{i=1}^{24} i = 4\left[\frac{24(25)}{2}\right] = 1200$

16. $\sum_{i=1}^{16}(5i - 4) = 5\sum_{i=1}^{16} i - 4(16) = 5\left[\frac{16(17)}{2}\right] - 64 = 616$

17. $\sum_{i=1}^{20}(i - 1)^2 = \sum_{i=1}^{19} i^2 = \left[\frac{19(20)(39)}{6}\right] = 2470$

18. $\sum_{i=1}^{10}(i^2 - 1) = \sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} 1 = \left[\frac{10(11)(21)}{6}\right] - 10 = 375$

19. $\sum_{i=1}^{15} i(i - 1)^2 = \sum_{i=1}^{15} i^3 - 2\sum_{i=1}^{15} i^2 + \sum_{i=1}^{15} i$

$= \frac{15^2(16)^2}{4} - 2\frac{15(16)(31)}{6} + \frac{15(16)}{2}$

$= 14{,}400 - 2480 + 120 = 12{,}040$

20. $\sum_{i=1}^{25}(i^3 - 2i) = \sum_{i=1}^{25} i^3 - 2\sum_{i=1}^{25} i$

$= \frac{(25)^2(26)^2}{4} - 2\frac{25(26)}{2}$

$= 105{,}625 - 650$

$= 104{,}975$