4.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Integration and Differentiation In Exercises 1 and 2, verify the statement by showing that the derivative of the right side equals the integrand of the left side.

1. $\int\left(-\frac{6}{x^4}\right)dx = \frac{2}{x^3} + C$

2. $\int\left(8x^3 + \frac{1}{2x^2}\right)dx = 2x^4 - \frac{1}{2x} + C$

Solving a Differential Equation In Exercises 3–6, find the general solution of the differential equation and check the result by differentiation.

3. $\frac{dy}{dt} = 9t^2$

4. $\frac{dy}{dt} = 5$

5. $\frac{dy}{dx} = x^{3/2}$

6. $\frac{dy}{dx} = 2x^{-3}$

Rewriting Before Integrating In Exercises 7–10, complete the table to find the indefinite integral.

Original Integral	Rewrite	Integrate	Simplify
7. $\int \sqrt[3]{x}\,dx$			
8. $\int \frac{1}{4x^2}\,dx$			
9. $\int \frac{1}{x\sqrt{x}}\,dx$			
10. $\int \frac{1}{(3x)^2}\,dx$			

Finding an Indefinite Integral In Exercises 11–32, find the indefinite integral and check the result by differentiation.

11. $\int (x + 7)\,dx$

12. $\int (13 - x)\,dx$

13. $\int (x^5 + 1)\,dx$

14. $\int (8x^3 - 9x^2 + 4)\,dx$

15. $\int (x^{3/2} + 2x + 1)\,dx$

16. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

17. $\int \sqrt[3]{x^2}\,dx$

18. $\int \left(\sqrt[4]{x^3} + 1\right)dx$

19. $\int \frac{1}{x^5}\,dx$

20. $\int \frac{3}{x^7}\,dx$

21. $\int \frac{x + 6}{\sqrt{x}}\,dx$

22. $\int \frac{x^4 - 3x^2 + 5}{x^4}\,dx$

23. $\int (x + 1)(3x - 2)\,dx$

24. $\int (4t^2 + 3)^2\,dt$

25. $\int (5\cos x + 4\sin x)\,dx$

26. $\int (t^2 - \cos t)\,dt$

27. $\int (1 - \csc t \cot t)\,dt$

28. $\int (\theta^2 + \sec^2\theta)\,d\theta$

29. $\int (\sec^2\theta - \sin\theta)\,d\theta$

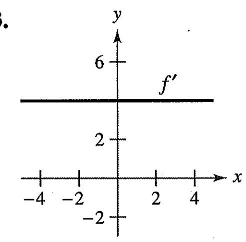
30. $\int \sec y(\tan y - \sec y)\,dy$

31. $\int (\tan^2 y + 1)\,dy$

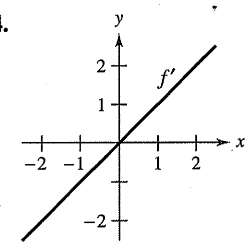
32. $\int (4x - \csc^2 x)\,dx$

Sketching a Graph In Exercises 33 and 34, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.) To print an enlarged copy of the graph, go to *MathGraphs.com.*

33.



34.



Finding a Particular Solution In Exercises 35–42, find the particular solution that satisfies the differential equation and the initial condition.

35. $f'(x) = 6x,\ f(0) = 8$

36. $g'(x) = 4x^2,\ g(-1) = 3$

37. $h'(t) = 8t^3 + 5,\ h(1) = -4$

38. $f'(s) = 10s - 12s^3,\ f(3) = 2$

39. $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$

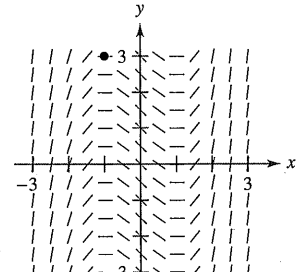
40. $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

41. $f''(x) = x^{-3/2},\ f'(4) = 2,\ f(0) = 0$

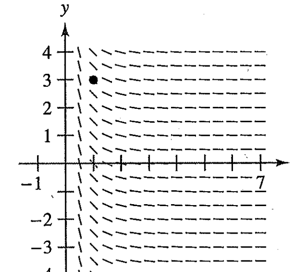
42. $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$

Slope Field In Exercises 43 and 44, a differential equation, a point, and a slope field are given. A *slope field* (or *direction field*) consists of line segments with slopes given by the differential equation. These line segments give a visual perspective of the slopes of the solutions of the differential equation. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the indicated point. (To print an enlarged copy of the graph, go to *MathGraphs.com.*) (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).

43. $\frac{dy}{dx} = x^2 - 1,\ (-1, 3)$



44. $\frac{dy}{dx} = -\frac{1}{x^2},\ x > 0,\ (1, 3)$



Slope Field **In Exercises 45 and 46, (a) use a graphing utility to graph a slope field for the differential equation, (b) use integration and the given point to find the particular solution of the differential equation, and (c) graph the solution and the slope field in the same viewing window.**

45. $\frac{dy}{dx} = 2x, (-2, -2)$

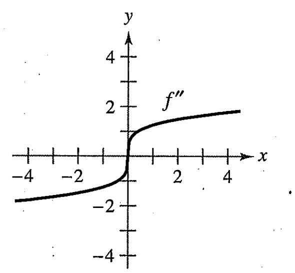
46. $\frac{dy}{dx} = 2\sqrt{x}, (4, 12)$

WRITING ABOUT CONCEPTS

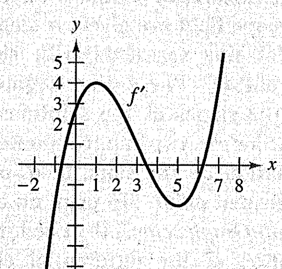
47. Antiderivatives and Indefinite Integrals What is the difference, if any, between finding the antiderivative of $f(x)$ and evaluating the integral $\int f(x)\,dx$?

48. Comparing Functions Consider $f(x) = \tan^2 x$ and $g(x) = \sec^2 x$. What do you notice about the derivatives of $f(x)$ and $g(x)$? What can you conclude about the relationship between $f(x)$ and $g(x)$?

49. Sketching Graphs The graphs of f and f' each pass through the origin. Use the graph of f'' shown in the figure to sketch the graphs of f and f'. To print an enlarged copy of the graph, go to *MathGraphs.com.*



50. HOW DO YOU SEE IT? Use the graph of f' shown in the figure to answer the following.



(a) Approximate the slope of f at $x = 4$. Explain.

(b) Is it possible that $f(2) = -1$? Explain.

(c) Is $f(5) - f(4) > 0$? Explain.

(d) Approximate the value of x where f is maximum. Explain.

(e) Approximate any open intervals in which the graph of f is concave upward and any open intervals in which it is concave downward. Approximate the x-coordinates of any points of inflection.

51. Tree Growth An evergreen nursery usually sells a certain type of shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by $dh/dt = 1.5t + 5$, where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

52. Population Growth The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days $(0 \le t \le 10)$. That is,

$$\frac{dP}{dt} = k\sqrt{t}.$$

The initial size of the population is 500. After 1 day the population has grown to 600. Estimate the population after 7 days.

Vertical Motion **In Exercises 53–55, use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)**

53. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

54. With what initial velocity must an object be thrown upward (from ground level) to reach the top of the Washington Monument (approximately 550 feet)?

55. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

(a) How many seconds after its release will the bag strike the ground?

(b) At what velocity will it hit the ground?

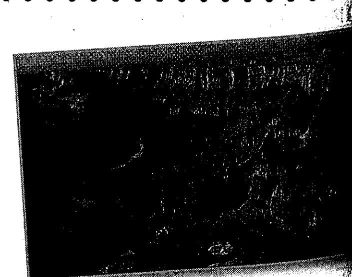
Vertical Motion **In Exercises 56–58, use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. (Neglect air resistance.)**

56. A baseball is thrown upward from a height of 2 meters with an initial velocity of 10 meters per second. Determine its maximum height.

57. With what initial velocity must an object be thrown upward (from a height of 2 meters) to reach a maximum height of 200 meters?

58. Grand Canyon

The Grand Canyon is 1800 meters deep at its deepest point. A rock is dropped from the rim above this point. Write the height of the rock as a function of the time t in seconds. How long will it take the rock to hit the canyon floor?



Josemaria Toscano/Shutterstock.com

59. Lunar Gravity On the moon, the acceleration due to gravity is -1.6 meters per second per second. A stone is dropped from a cliff on the moon and hits the surface of the moon 20 seconds later. How far did it fall? What was its velocity at impact?

60. Escape Velocity The minimum velocity required for an object to escape Earth's gravitational pull is obtained from the solution of the equation

$$\int v\,dv = -GM\int \frac{1}{y^2}\,dy$$

where v is the velocity of the object projected from Earth, y is the distance from the center of Earth, G is the gravitational constant, and M is the mass of Earth. Show that v and y are related by the equation

$$v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$$

where v_0 is the initial velocity of the object and R is the radius of Earth.

Rectilinear Motion In Exercises 61–64, consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.

61. $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals on which the particle is moving to the right.

(c) Find the velocity of the particle when the acceleration is 0.

62. Repeat Exercise 61 for the position function

$x(t) = (t - 1)(t - 3)^2, \quad 0 \le t \le 5.$

63. A particle moves along the x-axis at a velocity of $v(t) = 1/\sqrt{t}$, $t > 0$. At time $t = 1$, its position is $x = 4$. Find the acceleration and position functions for the particle.

64. A particle, initially at rest, moves along the x-axis such that its acceleration at time $t > 0$ is given by $a(t) = \cos t$. At the time $t = 0$, its position is $x = 3$.

(a) Find the velocity and position functions for the particle.

(b) Find the values of t for which the particle is at rest.

65. Acceleration The maker of an automobile advertises that it takes 13 seconds to accelerate from 25 kilometers per hour to 80 kilometers per hour. Assume the acceleration is constant.

(a) Find the acceleration in meters per second per second.

(b) Find the distance the car travels during the 13 seconds.

66. Deceleration A car traveling at 45 miles per hour is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied.

(a) How far has the car moved when its speed has been reduced to 30 miles per hour?

(b) How far has the car moved when its speed has been reduced to 15 miles per hour?

(c) Draw the real number line from 0 to 132. Plot the points found in parts (a) and (b). What can you conclude?

67. Acceleration At the instant the traffic light turns green, a car that has been waiting at an intersection starts with a constant acceleration of 6 feet per second per second. At the same instant, a truck traveling with a constant velocity of 30 feet per second passes the car.

(a) How far beyond its starting point will the car pass the truck?

(b) How fast will the car be traveling when it passes the truck?

68. Acceleration Assume that a fully loaded plane starting from rest has a constant acceleration while moving down a runway. The plane requires 0.7 mile of runway and a speed of 160 miles per hour in order to lift off. What is the plane's acceleration?

True or False? In Exercises 69–74, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

69. The antiderivative of $f(x)$ is unique.

70. Each antiderivative of an nth-degree polynomial function is an $(n + 1)$th-degree polynomial function.

71. If $p(x)$ is a polynomial function, then p has exactly one antiderivative whose graph contains the origin.

72. If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$, then

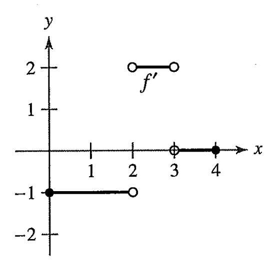
$F(x) = G(x) + C.$

73. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + C.$

74. $\int f(x)g(x)\,dx = \int f(x)\,dx \int g(x)\,dx$

75. Horizontal Tangent Find a function f such that the graph of f has a horizontal tangent at $(2, 0)$ and $f''(x) = 2x$.

76. Finding a Function The graph of f' is shown. Find and sketch the graph of f given that f is continuous and $f(0) = 1$.



77. Proof Let $s(x)$ and $c(x)$ be two functions satisfying $s'(x) = c(x)$ and $c'(x) = -s(x)$ for all x. If $s(0) = 0$ and $c(0) = 1$, prove that $[s(x)]^2 + [c(x)]^2 = 1$.

PUTNAM EXAM CHALLENGE

78. Suppose f and g are non-constant, differentiable, real-valued functions defined on $(-\infty, \infty)$. Furthermore, suppose that for each pair of real numbers x and y,

$$f(x + y) = f(x)f(y) - g(x)g(y) \quad \text{and}$$
$$g(x + y) = f(x)g(y) + g(x)f(y).$$

If $f'(0) = 0$, prove that $(f(x))^2 + (g(x))^2 = 1$ for all x.

This problem was composed by the Committee on the Putnam Prize Competition.