Calculus 4.1a Notes Antiderivative Formulas

If $f(x) = x^2$, *what is* $f'(x)$?

Using Power Rule, $\frac{d}{dx}u^n = n * u^{n-1}$, *we know that* $f'(x) = 2x$

To put the steps for Derivatives Power rule into words:

1) Bring exponent down in front of variable and ________________

2) ________________ exponent by 1

If $f'(x) = 2x$, *what steps can we take to find* $f(x)$?

We can "undo" the previous derivative steps:

1) ________________ 1 to the exponent

2) ________________ by the new exponent

Power Rule for Integration:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Antidifferentiation Notation:

NOTATION: $\int 2x\,dx = x^2 + C$

Integral integrand identifies the independent variable constant of integration

Consider the below functions:

$f(x) = x^2 + 5$
$f(x) = x^2 - 13$
$f(x) = x^2 + 126$

Since we can add a constant to any of these functions and still result in the same derivative, the **antiderivative** of a function will be in the form of $f(x) + C$ to show the family of functions that share the same derivative.

The process of integration is called **antidifferentiation** or taking the indefinite integral.

The indefinite integral results in a function.

The definite integral results in a number.

Integration Formulas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$
2. $\int a\, dx = ax + C$
3. $\int \frac{1}{u}\, du = \ln|u| + C$

Important: The derivative and integral are Inverse operations of each other.

4) $\int f'(x)dx = f(x) + C$

5) $\frac{d}{dx}[\int f(x)\, dx] = f(x)$

Recall Power Rule Conditions:

1) Rewrite as rational exponents 2) All variables in numerator 3) Expand expression fully

Class Examples:

1. $\int 7x\, dx =$
2. $\int 7x^3 dx =$
3. $\int 2x + 3x^2 - 5x^4\, dx =$
4. $\int (3x-1)^2 dx =$
5. $\int \frac{x+1}{\sqrt{x}} dx =$
6. $\int \frac{3}{y\sqrt{y}} dy =$
7. $\int \frac{3\sqrt{x}(1-x)^2}{\sqrt[3]{x}} dx =$

Key

Calculus 4.1a Notes Antiderivative Formulas

$If\ f(x) = x^2, what\ is\ f'(x)?$

$Using\ Power\ Rule,\ \frac{d}{dx}u^n = n * u^{n-1}, we\ know\ that\ f'(x) = 2x$

To put the steps for Derivatives Power rule into words:

1) Bring exponent down in front of variable and multiply
2) subtract exponent by 1

$If\ f'(x) = 2x, what\ steps\ can\ we\ take\ to\ find\ f(x)?$ $f(x) = \frac{2x^{1+1}}{2} = x^2$

We can "undo" the previous derivative steps:

1) Add 1 to the exponent
2) Divide by the new exponent

Power Rule for Integration:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Antidifferentiation Notation:

NOTATION: $\int 2x\,dx = x^2 + C$

Integral — integrand — identifies the independent variable — constant of integration

Consider the below functions:

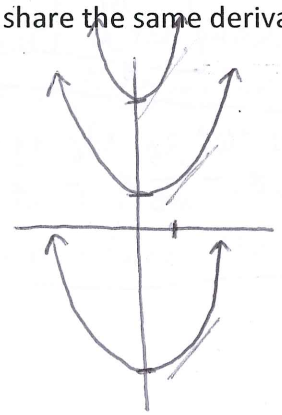
$$\left.\begin{aligned} f(x) &= x^2 + 5 \\ f(x) &= x^2 - 13 \\ f(x) &= x^2 + 126 \end{aligned}\right\} f'(x) = 2x$$

Since we can add a constant to any of these functions and still result in the same derivative, the **antiderivative** of a function will be in the form of $f(x) + C$ to show the family of functions that share the same derivative.

The process of integration is called **antidifferentiation** or taking the indefinite integral.

The indefinite integral results in a function.

The definite integral results in a number.



Integration Formulas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$

2. $\int a\, dx = ax + C$

3. $\int \frac{1}{u}\, du = \ln|u| + C$

Important: The derivative and integral are Inverse operations of each other.

4) $\int f'(x)dx = f(x) + C$

5) $\frac{d}{dx}[\int f(x)\, dx] = f(x)$

Recall Power Rule Conditions:

1) Rewrite as rational exponents 2)All variables in numerator 3) Expand expression fully

Class Examples:

1. $\int 7x\, dx = \boxed{\frac{7x^2}{2} + C}$

1b) $\int 7 dx = \boxed{7x + C}$

2. $\int 7x^3 dx = \boxed{\frac{7x^4}{4} + C}$

3. $\int 2x + 3x^2 - 5x^4\, dx =$

$\frac{2x^2}{2} + \frac{3x^3}{3} - \frac{5x^5}{5} + C$

$\boxed{x^2 + x^3 - x^5 + C}$

4. $\int (3x-1)^2 dx = \int (3x-1)(3x-1)dx$

$\int 9x^2 - 6x + 1\, dx$

$\frac{9x^3}{3} - \frac{6x^2}{2} + x + C$

$\boxed{3x^3 - 3x^2 + x + C}$

5. $\int \frac{x+1}{\sqrt{x}} dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} dx$

$\int x^{1/2} + x^{-1/2} dx$

$\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \boxed{\frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

6. $\int \frac{3}{y\sqrt{y}} dy = \int \frac{3}{y y^{1/2}} dy = \int \frac{3}{y^{3/2}} dy$

$\int 3y^{-3/2} dy = \frac{3y^{-1/2}}{-1/2} = \boxed{\frac{-6}{y^{1/2}} + C}$

7. $\int \frac{3\sqrt{x}(1-x)^2}{\sqrt[3]{x}} dx = \int 3x^{1/6}(1 - 2x + x^2) dx$

$\int \frac{3x^{1/2}(1 - 2x + x^2)}{x^{1/3}}$

$\int 3x^{1/6} - 6x^{7/6} + 3x^{13/6} dx$

$\frac{3x^{7/6}}{7/6} - \frac{6x^{13/6}}{13/6} + \frac{3x^{19/6}}{19/6} + C$

$\boxed{\frac{18}{7}x^{7/6} - \frac{36}{13}x^{13/6} + \frac{18}{19}x^{19/6} + C}$