4.1a Antiderivative Formulas

If $f(x) = x^2$, what is $f'(x)$?

Using power rule, $\frac{d}{dx} u^n = n \cdot u^{n-1}$, we know that $f'(x) = 2x$.

To put derivative power rule into words:

1) Bring exponent in front to multiply
2) Subtract exponent by 1.

So if $f'(x) = 2x$, what steps can we take to find $f(x)$?

We can "undo" the previous derivative steps.

1) Add 1 to exponent
2) Divide by new exponent

constant of integration

Power Rule for Integration: $\int \underbrace{u^n}_{\text{integrand}} \underbrace{du}_{\text{variable of integration}} = \frac{u^{n+1}}{n+1} + C$

Integral sign

integrand

variable of integration (identifies the independent variable)

Consider the below functions:

$$\left.\begin{aligned} f(x) &= x^2+5 \\ f(x) &= x^2-13 \\ f(x) &= x^2+126 \end{aligned}\right\} f'(x) = 2x$$

Since we can add a constant to any of these functions and still result in the same derivative, the antiderivative of a function will be in the form $f(x) + C$ to show the family of functions that share the same derivative

This process of integration is called antidifferentiation or taking the indefinite integral.

- The indefinite integral results in a function
- The definite integral results in a number.

4.1a (continued)

Integration Formulas:

1) $\int u^n du = \frac{u^{n+1}}{n+1} + c$

2) $\int a\, dx = ax + c$ (a as a constant)

3) $\int \frac{1}{u} du = \ln|u| + c$

Important: The derivative and integral are Inverse operations.

4) $\int f'(x)dx = f(x) + c$

5) $\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$

Ex. 1 $\int 7x\,dx = \boxed{\frac{7x^2}{2} + c}$

2) $\int 7x^3 dx = 7\int x^3 dx = 7\left(\frac{x^4}{4}\right) + c = \boxed{\frac{7x^4}{4} + c}$

3) $\int 2x + 3x^2 - 5x^4 dx = \frac{2x^2}{2} + \frac{3x^3}{3} - \frac{5x^5}{5} + c = \boxed{x^2 + x^3 - x^5 + c}$

4) $\int (3x-1)^2 dx = \int (3x-1)(3x-1)dx = \int 9x^2 - 6x + 1\,dx = \frac{9x^3}{3} - \frac{6x^2}{2} + x + c$

$= \boxed{3x^3 - 3x^2 + x + c}$

5) $\int \frac{x+1}{\sqrt{x}} dx$

$= \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} dx$

$= \int \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}} dx$

$= \int x^{1/2} + x^{-1/2} dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c$

$= \frac{2}{3}x^{3/2} + 2x^{1/2} + c$

or

$= \boxed{\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c}$

4.1a p.255-256 #1, 3, 9-33 odd, 43, 45

1) $\int \frac{-9}{x^4}dx = \int -9x^{-4}dx = -9\left(\frac{x^{-3}}{-3}\right)+c = \boxed{\frac{3}{x^3}+c}$

Be sure variable is a numerator before applying power rule.

3) $\int (x-2)(x+2)dx = \int x^2-4\,dx = \boxed{\frac{x^3}{3}-4x+c}$

9) $\int \sqrt[3]{x}\,dx = \int x^{1/3}dx = \frac{x^{1/3+3/3}}{4/3}+c = \frac{3}{4}x^{4/3}+c = \boxed{\frac{3\sqrt[3]{x^4}}{4}+c}$

11) $\int \frac{1}{x\sqrt{x}}dx = \int \frac{1}{x^1x^{1/2}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx = \frac{x^{-3/2+2/2}}{-1/2} = -2x^{-1/2}+c$

$= \boxed{\frac{-2}{\sqrt{x}}+c}$

13) $\int \frac{1}{2x^3}dx = \int \frac{1}{2}x^{-3}dx = \frac{1}{2}\left(\frac{x^{-2}}{-2}\right)+c = \boxed{-\frac{1}{4x^2}+c}$

27) $\int \frac{x^2+x+1}{\sqrt{x}}dx = \int \frac{x^2}{x^{1/2}}+\frac{x}{x^{1/2}}+\frac{1}{x^{1/2}}dx = \int x^{3/2}+x^{1/2}+x^{-1/2}dx$

$= \frac{x^{3/2+2/2}}{5/2}+\frac{x^{1/2+2/2}}{3/2}+\frac{x^{-1/2+2/2}}{1/2}+c = \frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2x^{1/2}+c$

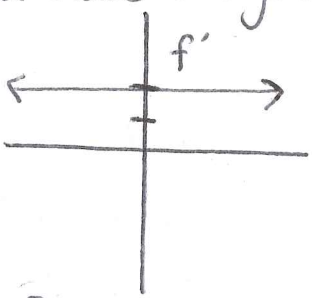
$= \boxed{\frac{2\sqrt{x^5}}{5}+\frac{2\sqrt{x^3}}{3}+2\sqrt{x}+c}$

31) $\int y^2\sqrt{y}\,dy = \int y^2 y^{1/2}\,dy = \int y^{5/2}\,dy = \dfrac{y^{5/2+2/2}}{7/2} + c$

$= \dfrac{2}{7}y^{7/2} + c = \boxed{\dfrac{2\sqrt{y^7}}{7} + c}$

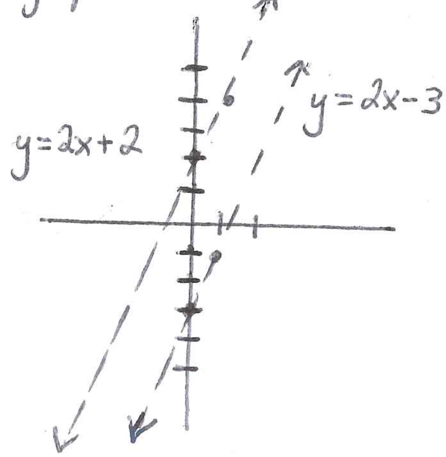
33) $\int 1\,dx = \boxed{x + c}$

43) Graph of the derivative is given. Sketch the graphs of two functions that have the given derivative.

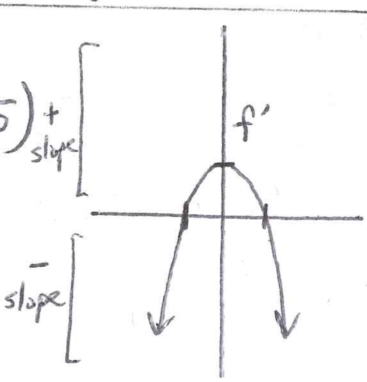


sketch graph with a constant slope of $m=2$

$\int 2\,dx = 2x + c$



45)



$f'(x) = 1 - x^2$

$\int 1 - x^2\,dx = x - \dfrac{x^3}{3} + c$