Calculus 4.1a Notes Antiderivative Formulas

If $f(x) = x^2$, *what is* $f'(x)$?

Using Power Rule, $\frac{d}{dx}u^n = n * u^{n-1}$, *we know that* $f'(x) = 2x$

To put the steps for Derivatives Power rule into words:

1) Bring exponent down in front of variable and ________________

2) ________________ exponent by 1

If $f'(x) = 2x$, *what steps can we take to find* $f(x)$?

We can "undo" the previous derivative steps:

1) ________________ 1 to the exponent

2) ________________ by the new exponent

Power Rule for Integration:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Antidifferentiation Notation:

NOTATION: $\int 2x\,dx = x^2 + C$

Integral — integrand — identifies the independent variable — constant of integration

Consider the below functions:

$f(x) = x^2 + 5$
$f(x) = x^2 - 13$
$f(x) = x^2 + 126$

Since we can add a constant to any of these functions and still result in the same derivative, the **antiderivative** of a function will be in the form of $f(x) + C$ to show the family of functions that share the same derivative.

The process of integration is called **antidifferentiation** or taking the indefinite integral.

The indefinite integral results in a function.

The definite integral results in a number.

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Integration Formulas

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C$
2. $\int a\, dx = ax + C$
3. $\int \frac{1}{u}\, du = \ln|u| + C$

Important: The derivative and integral are Inverse operations of each other.

4) $\int f'(x)dx = f(x) + C$

5) $\frac{d}{dx}[\int f(x)\, dx] = f(x)$

Recall Power Rule Conditions:

1) Rewrite as rational exponents 2) All variables in numerator 3) Expand expression fully

Class Examples:

1. $\int 7x\, dx =$

2. $\int 7x^3 dx =$

3. $\int 2x + 3x^2 - 5x^4\, dx =$

4. $\int (3x-1)^2 dx =$

5. $\int \frac{x+1}{\sqrt{x}}\, dx =$

6. $\int \frac{3}{y\sqrt{y}}\, dy =$

7. $\int \frac{3\sqrt{x}(1-x)^2}{\sqrt[3]{x}}\, dx =$

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Calculus **4.1b Notes** **Trig Integrals and Differential Equations**

Review Derivative Trig Rules:

1) $\frac{d}{dx}\sin u =$

3) $\frac{d}{dx}\cos u =$

2) $\frac{d}{dx}\tan u =$

4) $\frac{d}{dx}\cot u =$

5) $\frac{d}{dx}\sec u =$

6) $\frac{d}{dx}\csc u =$

Integral Trig Rules:

1) $\int \sin u\, du =$

2) $\int \cos u\, du =$

3) $\int \sec^2 u\, du =$

4) $\int \csc^2 u\, du =$

5) $\int \sec u \tan u\, du =$

6) $\int \csc u \cot u\, du =$

Classwork Examples:

1. $\int \frac{\tan x}{\cos x} - \sin x\ dx$

2. $\int \frac{\sin x}{\cos^2 x} dx$

3. $\int (1 + \cot^2 x) dx$

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<u>Differential Equations:</u> These are simply equations that involve derivatives.

<u>Steps for solving Differential equations:</u>

1. Rewrite y' as $\frac{dy}{dx}$
2. Separate variables on either side of equation
3. Take the integral of both sides

Solve for C if finding a specific solution/equation to the differential equation

Example 3: Suppose y' = 2. Solve for y.

Example 4: Solve this <u>General</u> Differential equation. $\frac{dy}{dx} = x^3$

Example 5: Solve this <u>Specific</u> differential equation: y' = 3x – 4 and the point (4, 10) is on the graph of y.

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x + 4$, $f'(0) = 3$, and $f(1) = 5$.
Find $f(x)$.

※ To help distinguish the constants of integration, use "+C" for the first constant and use "+K" for the second constant of integration.

Ex. 7 Given $g''(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

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4.1, 4.2, 4.6 Formula Sheet:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

Area using Limit Definition

$$\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(left\ endpoint + width * i)$$

width $= \frac{b-a}{n}$

Trapezoid Area:

$$Area = \frac{1}{2}w(h_1 + h_2)$$

4-2, 4-6 Riemann Sums WS: Using Tables of Values

1) Selected values of a function, *f*, are given in the table below:

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

a) Give the middle approximation with 3 subintervals for *f* on the interval [0, 20]

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

b) Use right-handed rectangles to approximate the area with 3 subintervals for *f* on the interval [0, 20]

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

c) Use left-handed rectangles to approximate the area with 3 subintervals for *f* on the interval [0, 9]

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

d) Use trapezoids to approximate the area with 2 subintervals for *f* on the interval [0, 20]

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2) Selected values of a function, *f*, are given in the table below:

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

a) Give the middle approximation with 2 subintervals for *f* on the interval [1, 20]

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

b) Use right-handed rectangles to approximate the area with 3 subintervals for *f* on the interval [3, 17]

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

c) Use left-handed rectangles to approximate the area with 4 subintervals for *f* on the interval [1, 12]

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

d) Use trapezoids to approximate the area with 3 subintervals for *f* on the interval [3, 17]

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AP Calculus AB 4-1,4-2, 4-6 Quiz Review #1

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}[(i+1)^2-(2-i)^3]$$

2. Use Sigma notation to write the sum: $\frac{2}{\sqrt[3]{5-2}}+\frac{4}{\sqrt[3]{5-4}}+\frac{6}{\sqrt[3]{5-6}}+\frac{8}{\sqrt[3]{5-8}}$

3. Use 3 middle rectangles to approximate the area of the region bounded by $f(x)=x^2+3$, the x-axis, $x=1$, and $x=6$.

4. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7]

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10]

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10]

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7]

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5. Given the region bounded by $g(x) = 6 - x^2$, the x-axis, $x = -1$, and $x = 2$. Use the limit definition to find the exact area of the region.

Find the most general antiderivative of $h(x)$. (Find $\int h(x)dx$)

6. $h(x) = 5x^4 - \pi + \frac{1}{2\sqrt{x}} + \frac{1}{3x^3}$

7. $h(x) = -2\cos x + 5\sin x - 5\csc x\cot x$

8. Find the most general expression of $f(x)$ if $f''(x) = 4x^3 - 5x^2 + 3x - 6$.

9. Find the specific expression of $f(x)$ if $f(x) = \int g(x)dx$, $g(x) = 3x^2 - 4x$, and $f(-1) = 2$

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AP Calculus AB 4-1, 4-2, 4-6 Quiz Review WS #2

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}[(i+1)^2 + 3(2i-1)^3]$$

2. Use Sigma notation to write the sum: $\frac{5-\sqrt{2}}{1}+\frac{5-\sqrt{4}}{4}+\frac{5-\sqrt{6}}{9}+\frac{5-\sqrt{8}}{16}$

3. Use 3 left rectangles to approximate the area of the region bounded by $f(x) = 1+2x^2$, the x-axis, $x=3$, and $x=7$.

4. Use the table of values on the right to estimate the below:

x	2	5	6	8	12	13	14
f(x)	1	2	8	3	1	6	5

a. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [5, 13]

b. Use 3 left-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [2, 14]

c. Use 2 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 14]

5. Given the region bounded by $g(x) = 3 + 2x^2$, the x-axis, $x = -2$, and $x = 1$. Use the **limit definition** to find the exact area of the region.

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Find the general antiderivative of g(x). (Find $\int g(x)dx$)

6. $g(x) = 3\cos x - 5\sin x + \csc x \cot x - 3\sqrt{x}$

7. $g(x) = \dfrac{2}{3(\sqrt[5]{x})} - 3x^2 - \dfrac{1}{3e^4}$

8. $g(x) = \dfrac{2x^3 - 5\sqrt{x} + 3(\sqrt[4]{x})}{x}$

9. Find the **general** expression of $f(x)$ if $f''(x) = 3x^3 + 5x^2 - x + 5$

10. Find the **specific** expression of $f(x)$ if $f'(x) = 5x^2 + 9x - 4$, $f(0) = 7$

AP Calculus AB 4-1, 4-2, 4-6 Morning Review WS #3

Calculators permitted.

1. Find the sum: $\sum_{i=1}^{3}\left[(2i+1)^{2}+(3i+1)^{3}\right]$

2. Use Sigma notation to write the sum: $\frac{7\sqrt{3}}{27}+\frac{7\sqrt{4}}{64}+\frac{7\sqrt{5}}{125}+\frac{7\sqrt{6}}{216}$

3. Use 4 middle rectangles to approximate the area of the region bounded by $f(x) = 3 + 2x^2$, the x-axis, $x = 1$, and $x = 7$.

4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [1, 15]

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [5, 11]

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 15]

5. Given the region bounded by $g(x) = 3 - 2x^2$, the x-axis, $x = -1$, and $x = 1$. Use the **limit definition** to find the exact area of the region.

Find the general antiderivative of $g(x)$. (Find $\int g(x)dx$)

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6. $g(x) = x(2x - 1)^2$

7. $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

8. $g(x) = \frac{x^3 - 2\sqrt{x} + \sqrt[4]{x}}{\sqrt{x}}$

9. Find the **general** expression of $f(x)$ if $f''(x) = 2x^3 + 3x^2 + x - 1$

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10. Find the **specific** expression of $f(x)$ if $f''(x) = 12x^2 + 18x - 4$, $f'(-1) = 9$, and $f(1) = 3$

AP Calculus AB 4-1, 4-2, 4-6 Quiz Review #1

Key

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}[(i+1)^2-(2-i)^3]$$

$= (2+1)^2-(2-2)^3+(3+1)^2-(2-3)^3+(4+1)^2-(2-4)^3$

$= 9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$

2. Use Sigma notation to write the sum: $\frac{2}{\sqrt[3]{5-2}}+\frac{4}{\sqrt[3]{5-4}}+\frac{6}{\sqrt[3]{5-6}}+\frac{8}{\sqrt[3]{5-8}}$

$$\sum_{i=1}^{4}\frac{2i}{\sqrt[3]{5-2i}}$$

3. Use 3 middle rectangles to approximate the area of the region bounded by $f(x)=x^2+3$, the x-axis, $x=1$, and $x=6$.

width $= \frac{b-a}{n} = \frac{6-1}{3} = \frac{5}{3}$

$1,\ 1\frac{5}{6}$ ... $2\frac{1}{6}$ ... $3\frac{1}{3}$ ... $3\frac{1}{6}$ ... 6

Area $= \frac{5}{3}\cdot f(1\frac{1}{6})+\frac{5}{3}f(2\frac{1}{6})+\frac{5}{3}(f(3\frac{1}{6}))$

$= \frac{5}{3}\cdot[(\frac{11}{6})^2+3]+\frac{5}{3}[(\frac{21}{6})^2+3]+\frac{5}{3}[(\frac{31}{6})^2+3]$

$= \frac{5}{3}(6.364)+\frac{5}{3}(15.25)+\frac{5}{3}(29.69) = \boxed{85.509}$

4. Use the table of values on the right to estimate the below.

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7].

Area $= 4\cdot f(0)+2f(4)+1\cdot f(6)$

Area $= 4(5)+2(3)+1(2)$

$= 20+6+2 = \boxed{28}$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10].

Area $= 6\cdot f(4)+4\cdot f(7)$

$= 6(3)+4(3)$

$= \boxed{30}$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10].

Area $= 2\cdot f(6)+1\cdot f(7)+3\cdot f(10)$

$= 2(2)+1(3)+3(5) = \boxed{22}$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7].

$A = \frac{1}{2}\cdot w\cdot[h_1+h_2]$

Area $= \frac{1}{2}\cdot 4[f(0)+f(4)]+\frac{1}{2}(2)[f(4)+f(6)]+\frac{1}{2}(1)[f(6)+f(7)]$

$A = 2(5+3)+1(3+2)+\frac{1}{2}(2+3)$

$\boxed{A = 23.5}$

5. Given the region bounded by $g(x)=6-x^2$, the x-axis, $x=-1$, and $x=2$. Use the limit definition to find the exact area of the region.

$w = \frac{2-(-1)}{n} = \frac{3}{n}$ $\quad A = \lim_{n\to\infty}\sum_{i=1}^{n}\text{width}\cdot f(a+\text{width}\cdot i)$

$A = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\cdot f(-1+\frac{3}{n}i)$

$A = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}[6-(-1+\frac{3}{n}i)^2]$

$= \lim_{n\to\infty}\frac{15}{n}(n)+\frac{18}{n^2}\cdot\frac{n(n+1)}{2}-\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$15+9-9 = \boxed{15}$

Find the most general antiderivative of $h(x)$. (Find $\int h(x)\,dx$)

6. $h(x)=5x^4-\pi+\frac{1}{2\sqrt{x}}+\frac{1}{3x^3}$

$\int 5x^4-\pi+\frac{1}{2}x^{-1/2}+\frac{1}{3}x^{-3}\,dx$

$\frac{5x^5}{5}-\pi x+\frac{1}{2}\cdot\frac{x^{1/2}}{1/2}+\frac{1}{3}\cdot\frac{x^{-2}}{-2}+C$

$\boxed{x^5-\pi x+x^{1/2}-\frac{1}{6x^2}+C}$

7. $h(x) = -2\cos x + 5\sin x - 5\csc x\cot x$

$\int -2\cos x+5\sin x-5\csc x\cot x\,dx$

$\boxed{-2\sin x-5\cos x+5\csc x+C}$

8. Find the most general expression of $f(x)$ if $f''(x)=4x^3-5x^2+3x-6$.

$f'(x)=\int 4x^3-5x^2+3x-6\,dx$

$f'(x)=\frac{4x^4}{4}-\frac{5x^3}{3}+\frac{3x^2}{2}-6x+C$

$f(x)=\int x^4-\frac{5}{3}x^3+\frac{3}{2}x^2-6x+C\,dx$

$\boxed{f(x)=\frac{1}{5}x^5-\frac{5x^4}{12}+\frac{1}{2}x^3-3x^2+Cx+K}$

9. Find the specific expression of $f(x)$ if $f(x)=\int g(x)\,dx$, $g(x)=3x^2-4x$, and $f(-1)=2$

$\int 3x^2-4x\,dx$

$f(x)=\frac{3x^3}{3}-\frac{4x^2}{2}+C$ (plug in (−1, 2) to solve for C)

$2=(-1)^3-2(-1)^2+C$

$2=-1-2+C$

$2=-3+C$

$5=C$

$\boxed{f(x)=x^3-2x^2+5}$

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Key

AP Calculus AB 4-1, 4-2, 4-6 Quiz Review WS #2

Calculators permitted.

1. Find the sum:

$$\sum_{i=2}^{4}[(i+1)^2+3(2i-1)^3] = (2+1)^2+3(2(2)-1)^3+(3+1)^2+3(2(3)-1)^3+(4+1)^2+3(2(4)-1)^3$$
$$= 9+3(27)+16+3(125)+25+(343)(3) = \boxed{1535}$$

2. Use Sigma notation to write the sum: $\frac{5-\sqrt{2}}{1}+\frac{5-\sqrt{4}}{4}+\frac{5-\sqrt{6}}{9}+\frac{5-\sqrt{8}}{16}$

$$\boxed{\sum_{i=1}^{4}\frac{5-\sqrt{2i}}{i^2}}$$

3. Use 3 left rectangles to approximate the area of the region bounded by $f(x)=1+2x^2$, the x-axis, $x=3$, and $x=7$.

$w=\frac{b-a}{n}=\frac{7-3}{3}=\frac{4}{3}$

Area $\approx \frac{4}{3}f(3)+\frac{4}{3}f(13/3)+\frac{4}{3}f(17/3)$
$= \frac{4}{3}(19)+\frac{4}{3}(38.55)+\frac{4}{3}(65.22) \approx \boxed{163.696}$

4. Use the table of values on the right to estimate the below.

x	2	5	6	8	12	13	14
f(x)	1	2	8	3	1	6	5

a. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [5, 13]

$3(8)+5(1)=\boxed{29}$

b. Use 3 left-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [2, 14]

$4(1)+6(8)+2(1)=\boxed{54}$

c. Use 2 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 14]

Area $=\frac{w}{2}[h_1+h_2]$

$\frac{6}{2}[8+1]+\frac{2}{2}[1+5] = 3(9)+1(6)=\boxed{33}$

5. Given the region bounded by $f(x)=3+2x^2$, the x-axis, $x=-2$, and $x=1$. Use the **limit definition** to find the exact area of the region.

width $=\frac{1-(-2)}{n}=\frac{3}{n}$

$$\lim_{n\to\infty}\frac{33n}{n}-\frac{72n^2}{2n^2}+\frac{108n^3}{6n^3}=33-36+18=\boxed{15}$$

Find the general antiderivative of $g(x)$. (Find $\int g(x)\,dx$)

6. $g(x)=3\cos x-5\sin x+\csc x\cot x-3\sqrt{x}$

$$\boxed{3\sin x+5\cos x-\csc x-2x^{3/2}+C}$$

7. $g(x)=\frac{2}{3(\sqrt[5]{x})}-3x^2-\frac{1}{3e^4}$

$$\boxed{\frac{5}{6}x^{4/5}-x^3-\frac{1}{3e^4}x+C}$$

8. $g(x)=\frac{2x^3-5\sqrt{x}+3(\sqrt[4]{x})}{x}$

$$\boxed{\frac{2}{3}x^3-10x^{1/2}+12x^{1/4}+C}$$

9. Find the **general** expression of $f(x)$ if $f''(x)=3x^3+5x^2-x+5$

$$\boxed{f(x)=\frac{3}{20}x^5+\frac{5}{12}x^4-\frac{1}{6}x^3+\frac{5}{2}x^2+Cx+K}$$

10. Find the **specific** expression of $f(x)$ if $f'(x)=5x^2+9x-4$, $f(0)=7$

$f(x)=\int 5x^2+9x-4\,dx$

$f(x)=\frac{5x^3}{3}+\frac{9x^2}{2}-4x+C$

$7=\frac{5}{3}(0)^3+\frac{9}{2}(0)^2-4(0)+C$

$C=7$

$$\boxed{f(x)=\frac{5}{3}x^3+\frac{9}{2}x^2-4x+7}$$

15 Key

AP Calculus AB 4-1, 4-2, 4-6 Morning Review WS #3

Calculators permitted.

1. Find the sum: $\sum_{i=1}^{3}\left[(2i+1)^2-(3i+1)^3\right]$ $= (2(1)+1)^2-(3(1)+1)^3+(2(2)+1)^2-(3(2)+1)^3+(2(3)+1)^2-(3(3)+1)^3$

$= 3^2-4^3+5^2-7^3+7^2-10^3 = \boxed{-1324}$

2. Use Sigma notation to write the sum: $\frac{7\sqrt{3}}{27}+\frac{7\sqrt{4}}{64}+\frac{7\sqrt{5}}{125}+\frac{7\sqrt{6}}{216}$

3. Use 4 middle rectangles to approximate the area of the region bounded by $f(x) = 3 + 2x^2$, the x-axis, $x = 1$, and $x = 7$.

$w = \frac{7-1}{4} = \frac{6}{4} = \frac{3}{2}$

1, 7/4, 5/2, 13/4, 8/2, 19/4, 11/2, 25/4, 14/2

Area $\approx \frac{3}{2}\cdot f(7/4)+\frac{3}{2}\cdot f(13/4)+\frac{3}{2}\cdot f(\frac{19}{4})+\frac{3}{2}f(\frac{25}{4})$

$= \frac{3}{2}(9.125)+\frac{3}{2}(24.125)+\frac{3}{2}(48.125)+\frac{3}{2}(81.125) = \boxed{243.75}$

4. Use the table of values on the right to estimate the below:

x	1	5	6	8	11	13	15
f(x)	4	2	7	3	1	6	5

a. Use 3 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [1, 15]

(widths 5, 5, 4)

1	5	6	8	11	13	15
4	2	7	3	1	6	5

$5(2)+5(3)+4(6) = \boxed{49}$

b. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [5, 11]

(widths 1, 2, 3)

5	6	8	11
2	7	3	1

$1(7)+2(3)+3(1) = \boxed{16}$

c. Use 4 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [6, 15]

*Area $= \frac{W}{2}[h_1+h_2]$

(widths 2, 3, 2, 2)

6	8	11	13	15
7	3	1	6	5

$\frac{2}{2}[7+3]+\frac{3}{2}[3+1]+\frac{2}{2}[1+6]+\frac{2}{2}[6+5]$

$10+\frac{3}{2}(4)+1(7)+1(11) = \boxed{34}$

5. Given the region bounded by $g(x) = 3 - 2x^2$, the x-axis, $x = -1$, and $x = 1$. Use the **limit definition** to find the exact area of the region.

$w = \frac{1-(-1)}{n} = \frac{2}{n}$

Area $= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\cdot f\left[-1+\frac{2}{n}i\right]$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\cdot\left[3-2(-1+\frac{2}{n}i)^2\right]$

$\sum_{i=1}^{n}\frac{2}{n}\left[3-2(1-\frac{4}{n}i+\frac{4}{n^2}i^2)\right]$

$\frac{2}{n}\left[3-2+\frac{8}{n}i-\frac{8}{n^2}i^2\right]$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left[1+\frac{8}{n}i-\frac{8}{n^2}i^2\right]$

$\sum_{i=1}^{n}\frac{2}{n}+\frac{16}{n^2}i-\frac{16}{n^3}i^2$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}+\sum\frac{16}{n^2}i-\sum\frac{16}{n^3}i^2$

$\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}1+\frac{16}{n^2}\sum_{i=1}^{n}i-\frac{16}{n^3}\sum_{i=1}^{n}i^2$

$\lim_{n\to\infty}\frac{2}{n}(n)+\frac{16}{n^2}\cdot\frac{n(n+1)}{2}-\frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty}\frac{2n}{n}+\frac{16n^2}{2n^2}-\frac{32n^3}{6n^3}$

$2+8-\frac{32}{6} = \boxed{\frac{14}{3}}$

(16)

Find the general antiderivative of g(x). (Find $\int g(x)dx$)

6. $g(x) = x(2x-1)^2$

$\int x(2x-1)^2 dx$

$\int x(2x-1)(2x-1)dx$

$\int x(4x^2-4x+1)dx$

$\int 4x^3-4x^2+x\,dx$

$\frac{4x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + C$

$x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} + C$

7. $g(x) = \frac{4}{\sqrt[3]{x}} - \sqrt{x} + 3x^2 - \frac{1}{3x^4}$

$\int 4x^{-1/3} - x^{1/2} + 3x^2 - \frac{1}{3}x^{-4}dx$

$4\left(\frac{x^{2/3}}{2/3}\right) - \frac{x^{3/2}}{3/2} + \frac{3x^3}{3} - \frac{1}{3}\cdot\frac{x^{-3}}{-3} + C$

$6x^{2/3} - \frac{2}{3}x^{3/2} + x^3 + \frac{1}{9x^3} + C$

8. $g(x) = \frac{x^3-2\sqrt{x}+\sqrt[4]{x}}{\sqrt{x}}$

$\int (x^3-2x^{1/2}+x^{1/4})x^{-1/2}dx$

$\int x^{5/2} - 2 + x^{-1/4}dx$

$\frac{x^{7/2}}{7/2} - 2x + \frac{x^{3/4}}{3/4} + C$

$\frac{2}{7}x^{7/2} - 2x + \frac{4}{3}x^{3/4} + C$

9. Find the **general** expression of $f(x)$ if $f''(x) = 2x^3 + 3x^2 + x - 1$

$f'(x) = \int 2x^3+3x^2+x-1\,dx$

$f'(x) = \frac{2x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2} - x + c$

$f(x) = \frac{1}{2}\cdot\frac{x^5}{5} + \frac{x^4}{4} + \frac{1}{2}\cdot\frac{x^3}{3} - \frac{x^2}{2} + Cx + k$

$f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{1}{2}x^2 + Cx + k$

10. Find the **specific** expression of $f(x)$ if $f''(x) = 12x^2 + 18x - 4$, $f'(-1) = 9$, and $f(1) = 3$

$f'(x) = \int 12x^2+18x-4\,dx$

$f'(x) = \frac{12x^3}{3} + \frac{18x^2}{2} - 4x + C$

$9 = 4(-1)^3 + 9(-1)^2 - 4(-1) + C$

$9 = -4 + 9 + 4 + C$

$0 = C$

$f'(x) = 4x^3 + 9x^2 - 4x$

$f(x) = \int 4x^3 + 9x^2 - 4x\,dx$

$f(x) = \frac{4x^4}{4} + \frac{9x^3}{3} - \frac{4x^2}{2} + k$

$f(x) = x^4 + 3x^3 - 2x^2 + k$

$3 = (1)^4 + 3(1)^3 - 2(1)^2 + k$

$3 = 1 + 3 - 2 + k$

$1 = k$

$f(x) = x^4 + 3x^3 - 2x^2 + 1$