Calculus **4.1b Notes** **Trig Integrals and Differential Equations**

Review Derivative Trig Rules:

1) $\frac{d}{dx}\sin u =$

3) $\frac{d}{dx}\cos u =$

2) $\frac{d}{dx}\tan u =$

4) $\frac{d}{dx}\cot u =$

5) $\frac{d}{dx}\sec u =$

6) $\frac{d}{dx}\csc u =$

Integral Trig Rules:

1) $\int \sin u\, du =$

2) $\int \cos u\, du =$

3) $\int \sec^2 u\, du =$

4) $\int \csc^2 u\, du =$

5) $\int \sec u \tan u\, du =$

6) $\int \csc u \cot u\, du =$

Classwork Examples:

1. $\int \frac{\tan x}{\cos x} - \sin x \ dx$

2. $\int \frac{\sin x}{\cos^2 x} dx$

3. $\int (1 + \cot^2 x) dx$

Differential Equations: These are simply equations that involve derivatives.

Steps for solving Differential equations:

1. Rewrite y' as $\frac{dy}{dx}$
2. Separate variables on either side of equation
3. Take the integral of both sides

Solve for C if finding a specific solution/equation to the differential equation

Example 3: Suppose y' = 2. Solve for y.

Example 4: Solve this General Differential equation. $\frac{dy}{dx} = x^3$

Example 5: Solve this Specific differential equation: y' = 3x – 4 and the point (4, 10) is on the graph of y.

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x + 4$, $f'(0) = 3$, and $f(1) = 5$.
Find $f(x)$.

* To help distinguish the constants of integration, use "$+C$" for the first constant and use "$+K$" for the second constant of integration.

Ex. 7 Given $g''(x) = 12x + 6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

Calculus **4.1b Notes** **Trig Integrals and Differential Equations** Key

Review Derivative Trig Rules:

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$

3) $\frac{d}{dx}\cos u = -\sin u \cdot u'$

2) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$

4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$

6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

Integral Trig Rules:

1) $\int \sin u\, du = -\cos u + C$

2) $\int \cos u\, du = \sin u + C$

3) $\int \sec^2 u\, du = \tan u + C$

4) $\int \csc^2 u\, du = -\cot u + C$

5) $\int \sec u \tan u\, du = \sec u + C$

6) $\int \csc u \cot u\, du = -\csc u + C$

Classwork Examples:

1. $\int \frac{\tan x}{\cos x} - \sin x \; dx$

$\int \sec x \tan x - \sin x\, dx$

$\sec x - (-\cos x) + C$

$\boxed{\sec x + \cos x + C}$

2. $\int \frac{\sin x}{\cos^2 x} dx$

$\int \frac{\sin x}{\cos x \cdot \cos x} \cdot dx$

$\int \tan x \sec x\, dx$

$\boxed{\sec x + C}$

3. $\int (1 + \cot^2 x) dx$ * think about trig identity $1+\cot^2 x = \csc^2 x$

$\int \csc^2 x\, dx = \boxed{-\cot x + C}$

Differential Equations: These are simply equations that involve derivatives.

Steps for solving Differential equations:

1. Rewrite y' as $\frac{dy}{dx}$
2. Separate variables on either side of equation
3. Take the integral of both sides

Solve for C if finding a specific solution/equation to the differential equation

Example 3: Suppose y' = 2. Solve for y.

$$\frac{dy}{dx} = 2$$

$$dy = 2dx$$

$$\int dy = \int 2\,dx$$

$$y = 2x + C$$

* we only need to add "+C" once on the right side of equation.

Example 4: Solve this General Differential equation. $\frac{dy}{dx} = x^3$

$$\frac{dy}{dx} = x^3$$

$$dy = x^3 dx$$

$$\int dy = \int x^3 dx$$

$$\boxed{y = \frac{x^4}{4} + C}$$

Example 5: Solve this Specific differential equation: y' = 3x – 4 and the point (4, 10) is on the graph of y.

$$y' = 3x - 4$$

$$\frac{dy}{dx} = 3x - 4$$

$$dy = (3x-4)dx$$

$$\int dy = \int 3x - 4\,dx$$

$$y = \frac{3x^2}{2} - 4x + C$$

plug in (4, 10)

$$10 = \frac{3}{2}(4)^2 - 4(4) + C$$

$$10 = 24 - 16 + C$$

$$10 = 8 + C$$

$$C = 2$$

$$\boxed{y = \frac{3}{2}x^2 - 4x + 2}$$

4.1b (continued) More diff. equation examples

Ex. 6 Suppose $f''(x) = 6x+4$, $f'(0) = 3$, and $f(1) = 5$. Find $f(x)$.

* To help distinguish the constants of integration, use "+c" for the first constant and use "+k" for the second constant of integration.

$f''(x) = 6x+4$

$f'(x) = \frac{6x^2}{2} + 4x + c$ use $f'(0)=3$ to solve for c.

$3 = \frac{6(0)^2}{2} + 4(0) + c$

$\underline{3 = c}$ $\rightarrow f'(x) = 3x^2 + 4x + 3$

$f'(x) = 3x^2 + 4x + 3$

$f(x) = \frac{3x^3}{3} + \frac{4x^2}{2} + 3x + k$ Use $f(1)=5$ to find k.

$f(x) = x^3 + 2x^2 + 3x + k$

$5 = 1^3 + 2(1)^2 + 3(1) + k$

$5 = 6 + k$

$\underline{-1 = k}$ $\longrightarrow$ $\boxed{f(x) = x^3 + 2x^2 + 3x - 1}$

Ex. 7 Given $g''(x) = 12x+6$ and $g(0) = 4$ and $g(1) = -2$. Find $g(x)$.

$g'(x) = \frac{12x^2}{2} + 6x + c$

$g(x) = \frac{6x^3}{3} + \frac{6x^2}{2} + cx + k$

$g(x) = 2x^3 + 3x^2 + cx + k$

$4 = 2(0)^3 + 3(0)^2 + c(0) + k$

$\underline{4 = k}$

$g(x) = 2x^3 + 3x^2 + cx + 4$

$-2 = 2(1)^3 + 3(1)^2 + c(1) + 4$

$-2 = 2 + 3 + c + 4$

$-2 = 9 + c$

$\underline{-11 = c}$

$\boxed{g(x) = 2x^3 + 3x^2 - 11x + 4}$