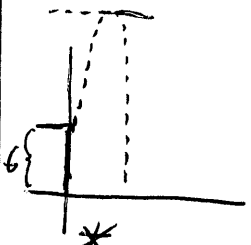
2017

Key

Ch. 4.1b Vertical Motion (Particle Motion) Homework pg. 251-252 #53, 55,56 57,58

Vertical Motion In Exercises 53–55, use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)



$s(0)=6$

53. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

$V(0)=60$ ft/s

* ball reaches highest point when $v(t)=0$

$a(t)=-32$

$v(t)=\int -32\,dt$

$v(t)=-32t+C$

$60=-32(0)+C$

$60=C$

$v(t)=-32t+60$

$s(t)=\int -32t+60\,dt$

$s(t)=\frac{-32t^2}{2}+60t+C$

$s(t)=-16t^2+60t+C$ ← $s(0)=6$

$6=-16(0)^2+60(0)+C$

$6=C$

$s(t)=-16t^2+60t+6$

$v(t)=-32t+60$

$0=-32t+60$

$32t=60$ $\quad t=\frac{60}{32}=\frac{15}{8}$ sec.

$s\left(\frac{15}{8}\right)=-16\left(\frac{15}{8}\right)^2+60\left(\frac{15}{8}\right)+6$

$s\left(\frac{15}{8}\right)=62.25$ feet

55. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

(a) How many seconds after its release will the bag strike the ground?

(b) At what velocity will it hit the ground?

$a(t)=-32$

$V_0=16$ ft/s $\quad v(0)=16$

$s_0=64$ ft. $\quad s(0)=64$

$a(t)=-32$

$v(t)=\int -32\,dt$ ← plug in $v(0)=16$

$v(t)=-32t+C$

$16=-32(0)+C$

$16=C$

$v(t)=-32t+16$

$s(t)=\int -32t+16\,dt=\frac{-32t^2}{2}+16t+C$ ← plug in $s(0)=64$

$s(t)=-16t^2+16t+C$

$64=-16(0)^2+16(0)+C$

$64=C$

$s(t)=-16t^2+16t+64$

* bag strikes the ground when $s(t)=0$

$0=-16t^2+16t+64$

$0=-16(t^2-t-4)$

$t=\frac{1\pm\sqrt{1^2-4(1)(-4)}}{2(1)}$

$\frac{1\pm\sqrt{17}}{2}\rightarrow\frac{1+\sqrt{17}}{2}\approx$ **2.562 sec** a)

b) $v(t)=-32t+16$

$v(2.562)=-32(2.562)+16$

$=$ **-65.9 ft/s**

Vertical Motion **In Exercises 56–58, use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. (Neglect air resistance.)**

56. A baseball is thrown upward from a height of 2 meters with an initial velocity of 10 meters per second. Determine its maximum height.

$s(0) = 2\text{ m}$ $\quad v(0) = 10\text{ m/s}$

$a(t) = -9.8$

$v(t) = \int -9.8\,dt$

$v(t) = -9.8t + C$

$10 = -9.8(0) + C \quad C = 10$

$v(t) = -9.8t + 10$

$s(t) = \int -9.8t + 10\,dt$

$s(t) = \frac{-9.8t^2}{2} + 10t + C$

$s(t) = -4.9t^2 + 10t + C$

plug in $s(0) = 2$

$s(t) = -4.9t^2 + 10t + C$

$2 = -4.9(0)^2 + 10(0) + C$

$2 = C$

$s(t) = -4.9t^2 + 10t + 2$

* Max height occurs when $v(t) = 0$

$v(t) = -9.8t + 10$

$0 = -9.8t + 10$

$9.8t = 10 \quad t \approx 1.02$ sec.

$s(1.02) = -4.9(1.02)^2 + 10(1.02) + 2$

$s(1.02) \approx 7.102\text{m}$ Max height

57. With what initial velocity must an object be thrown upward (from a height of 2 meters) to reach a maximum height of 200 meters? $s(0) = 2$

$s(t) = \frac{-9.8t^2}{2} + v_0 t + 2$

$s(t) = -4.9t^2 + v_0 t + 2$

$v(t) = -9.8t + v_0$

$0 = -9.8t + v_0$

$9.8t = v_0$

$t = \frac{v_0}{9.8}$

* set $s(t) =$ max height

$-4.9t^2 + v_0 t + 2 = 200$

$-4.9\left(\frac{v_0}{9.8}\right)^2 + v_0\left(\frac{v_0}{9.8}\right) + 2 = 200$

$\frac{-4.9v_0^2}{9.8^2} + \frac{v_0^2}{9.8} = 198$

$-4.9v_0^2 + 9.8v_0^2 = 198(9.8)^2$

$4.9v_0^2 = (9.8)^2 198$

$v_0 = 62.3\text{ m/s}$

• 58. Grand Canyon •

The Grand Canyon is 1800 meters deep at its deepest point. A rock is dropped from the rim above this point. Write the height of the rock as a function of the time t in seconds. How long will it take the rock to hit the canyon floor?

$v_0 = 0$

set $s(t) = 0$

$s(t) = -4.9t^2 + v_0 t + 1800$

$s(t) = -4.9t^2 + 0t + 1800$

$s(t) = -4.9t^2 + 1800$

$0 = -4.9t^2 + 1800$

$4.9t^2 = 1800$

$t^2 = \frac{1800}{4.9}$

$t \approx 9.2\text{ sec}$

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