Key

4.21 Solving Trig Equations Worksheet Date: _____

Find all solutions to the equation in the interval $[0, 2\pi)$. Do not use a calculator.

1. $2\cos x \sin x - \cos x = 0$

$\cos x$ $\cos x$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ | $2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $2\sin x = 1$

$\sin x = 1/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

2. $\sqrt{2}\tan x \cos x - \tan x = 0$

$\tan x$ $\tan x$

$\tan x(\sqrt{2}\cos x - 1) = 0$

$\tan x = 0$ | $\sqrt{2}\cos x - 1 = 0$

$x = 0, \pi$ | $\cos x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, 7\pi/4$

3. $\tan x \sin^2 x = \tan x$

$\tan x \sin^2 x - \tan x = 0$

$\tan x$ $\tan x$

$\tan x(\sin^2 x - 1) = 0$

$\tan x = 0$ | $\sin^2 x - 1 = 0$

$x = 0, \pi$ | $\sqrt{\sin^2 x} = \pm\sqrt{1}$

$\sin x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

4. $\sin x \tan^2 x = \sin x$

$\sin x \tan^2 x - \sin x = 0$

$\sin x$ $\sin x$

$\sin x(\tan^2 x - 1) = 0$

$\sin x = 0$ | $\tan^2 x - 1 = 0$

$x = 0, \pi$ | $\sqrt{\tan^2 x} = \sqrt{1}$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. $\tan^2 x = 3$

$\sqrt{\tan^2 x} = \pm\sqrt{3}$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

6. $2\sin^2 x = 1$

$\frac{2}{2}$ $\frac{1}{2}$

$\sqrt{\sin^2 x} = \pm\sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{\sqrt{1}}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Find all solutions to the equation. Do not use a calculator.

7. $4\cos^2 x - 4\cos x + 1 = 0$

* factor $4x^2 - 4x + 1 = 0$

$(x - \frac{1}{2})(x - \frac{1}{2})$

$(2x - 1)(2x - 1)$

$(2\cos x - 1)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ | $2\cos x - 1 = 0$, $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n,\ n \in \mathbb{Z}$

8. $2\sin^2 x + 3\sin x + 1 = 0$

* factor $2x^2 + 3x + 1 = 0$

$(x+1)(x+\frac{1}{2}) \rightarrow (x+1)(2x+1)$

$(\sin x + 1)(2\sin x + 1) = 0$

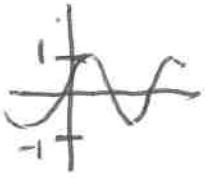
$\sin x + 1 = 0$ | $2\sin x + 1 = 0$

$\sin x = -1$ | $\sin x = -\frac{1}{2}$

$x = \frac{3\pi}{2}$ | $x = \frac{7\pi}{6}, \frac{11\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$

9. $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$



$\sin x = 0$ | $\sin x - 2 = 0$

$x = 0\pi$ | $\sin x = 2$

undefined

$+2\pi n,\ n \in \mathbb{Z}$

10. $3\sin x = 2\cos^2 x$

* $\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$3\sin x = 2(1 - \sin^2 x)$

$3\sin x = 2 - 2\sin^2 x$

$2\sin^2 x + 3\sin x - 2 = 0$

* factor $2x^2 + 3x - 2 = 0$

$(x+2)(x-\frac{1}{2})$

$(x+2)(2x-1)$

$(\sin x + 2)(2\sin x - 1) = 0$

$\sin x + 2 = 0$ | $2\sin x - 1 = 0$

$\sin x = -2$ | $\sin x = \frac{1}{2}$

undefined | $x = \frac{\pi}{6}, \frac{5\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$

11. $2\sin^2 x + 3\sin x = 2$

$2\sin^2 x + 3\sin x - 2 = 0$

* factor $2x^2 + 3x - 2 = 0$

$(x+2)(x-\frac{1}{2})$

$(x+2)(2x-1)$

$(\sin x + 2)(2\sin x - 1) = 0$

$\sin x = -2$ | $\sin x = \frac{1}{2}$

undefined | $x = \frac{\pi}{6}, 5\pi/6 + 2\pi n,\ n \in \mathbb{Z}$

12. $\sin^4 x + 2\sin^2 x - 3 = 0$

* factor $x^4 + 2x^2 - 3 = 0$

$(x^2 + 3)(x^2 - 1) = 0$

$(\sin^2 x + 3)(\sin^2 x - 1) = 0$

$\sin^2 x + 3 = 0$ | $\sin^2 x - 1 = 0$

$\sqrt{\sin^2 x} = \sqrt{-3}$ | $\sqrt{\sin^2 x} = \pm\sqrt{1}$

undefined | $\sin x = \pm 1$

$x = \pi/2, 3\pi/2 + 2\pi n,\ n \in \mathbb{Z}$