Key

4.22 Solving Trig Equations with Double Angles Worksheet Date: _____

Solve the equation for the variable on the interval $[0, 2\pi)$.

1. $\sqrt{2}\tan x = 2\sin x$

$\sqrt{2}\tan x - 2\sin x = 0$

$\frac{\sqrt{2}\sin x}{\cos x} - 2\sin x = 0$

$\sin x\left(\frac{\sqrt{2}}{\cos x} - 2\right) = 0$

$\sin x = 0$ | $\frac{\sqrt{2}}{\cos x} - 2 = 0$

$x = 0, \pi$ | $\frac{\sqrt{2}}{\cos x} = \frac{2}{1}$

$2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \pi/4, \frac{7\pi}{4}$

2. $3\tan^2 y - 1 = 0$

$3\tan^2 y = 1$

$\tan^2 y = \frac{1}{3}$

$\sqrt{\tan^2 y} = \pm\sqrt{\frac{1}{3}}$

$\tan y = \pm\frac{1}{\sqrt{3}}$

$y = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3. $\cos^3\theta = \cos\theta$

$\cos^3\theta - \cos\theta = 0$

$\cos\theta \quad \cos\theta$

$\cos\theta(\cos^2\theta - 1) = 0$

$\cos\theta = 0$ | $\cos^2\theta - 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\sqrt{\cos^2\theta} = \pm\sqrt{1}$

$\cos\theta = \pm 1$

$\theta = 0, \pi$

4. $5\sin\theta - 2\cos^2\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$5\sin\theta - 2(1 - \sin^2\theta) - 1 = 0$

$5\sin\theta - 2 + 2\sin^2\theta - 1 = 0$

$2\sin^2\theta + 5\sin\theta - 3 = 0$

*factor $2x^2 + 5x - 3 = 0$ (a, b, c)

$a \cdot c = -6$: $6, -1$; $2, -2$; sum $= b = 5$

$(x+3)(x-\frac{1}{2}) = 0$

$(x+3)(2x-1) = 0$

$(\sin\theta + 3)(2\sin\theta - 1) = 0$

$\sin\theta + 3 = 0$ | $2\sin\theta - 1 = 0$

$\sin\theta = -3$ | $\sin\theta = 1/2$

$\downarrow$ undefined

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

5. $\cos 2y = \frac{1}{2}$

$\downarrow$

$2\cos^2 y - 1 = \frac{1}{2}$

$2\cos^2 y = 1 + \frac{1}{2}$

$\frac{1}{2} \cdot 2\cos^2 y = \frac{3}{2} \cdot \frac{1}{2}$

$\cos^2 y = \frac{3}{4}$

$\sqrt{\cos^2 y} = \pm\sqrt{\frac{3}{4}}$

$\cos y = \pm\frac{\sqrt{3}}{\sqrt{4}}$

$\cos y = \pm\frac{\sqrt{3}}{2}$

$y = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

6. $3\tan^4 x - 10\tan^2 x + 3 = 0$

factor

*$3x^4 - 10x^2 + 3 = 0$

$a \cdot c = 9$: $-9, -1$; $3, -3$; sum $= b = -10$

$(x^2 - 3)(x^2 - \frac{1}{3})$

$(x^2 - 3)(3x^2 - 1)$

$(\tan^2 x - 3)(3\tan^2 x - 1) = 0$

$\tan^2 x - 3 = 0$ | $3\tan^2 x - 1 = 0$

$\tan^2 x = 3$ | $\tan^2 x = \frac{1}{3}$

$\sqrt{\tan^2 x} = \pm\sqrt{3}$ | $\sqrt{\tan^2 x} = \pm\sqrt{\frac{1}{3}}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ | $\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

Solve the equation for all values of the variable.

7. $\sin 2x = 2\sin x$

$2\sin x\cos x = 2\sin x$

$\frac{2\sin x\cos x}{2\sin x} - \frac{2\sin x}{2\sin x} = 0$

$2\sin x(\cos x - 1) = 0$

$2\sin x = 0$	$\cos x - 1 = 0$
$\sin x = 0$	$\cos x = 1$
$x = 0, \pi$	$x = 0 + 2\pi n,\ n \in \mathbb{Z}$

8. $\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x$

$1 - 2\sin^2 x = 3\sin x - 1$

$0 = 2\sin^2 x + 3\sin x - 2$

factor $2x^2 + 3x - 2 = 0$

a·c = −4; −1, 4; 2, 3; 6

$(x - \frac{1}{2})(x + 2) = 0$

$(2x - 1)(x + 2) = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$2\sin x - 1 = 0$	$\sin x + 2 = 0$
$\sin x = 1/2$	$\sin x = -2$
$x = \frac{\pi}{6}, \frac{5\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$	undefined

9. $\sin 2x - \tan x = 0$

$2\sin x\cos x - \tan x = 0$

$2\sin x\cos x - \frac{\sin x}{\cos x} = 0$

$\sin x\left(2\cos x - \frac{1}{\cos x}\right) = 0$

$\sin x = 0$	$2\cos x - \frac{1}{\cos x} = 0$
$x = 0, \pi$	$\frac{2\cos x}{1} = \frac{1}{\cos x}$
	$2\cos^2 x = 1$
	$\cos^2 x = 1/2$
	$\sqrt{\cos^2 x} = \pm\sqrt{1/2}$
	$\cos x = \pm\frac{1}{\sqrt{2}}$
	$\cos x = \pm\frac{\sqrt{2}}{2}$
	$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} + 2\pi n,\ n \in \mathbb{Z}$

10. $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

factor $2x^2 + x - 1 = 0$

a·c = −2; 2, −1; 2, 2; 4

$(x + 1)(x - 1/2)$

$(\cos x + 1)(2\cos x - 1) = 0$

$\cos x + 1 = 0$	$2\cos x - 1 = 0$
$\cos x = -1$	$\cos x = 1/2$
$x = \pi$	$x = \pi/3, 5\pi/3$
	$+ 2\pi n,\ n \in \mathbb{Z}$

11. $\cos x + \cos 3x = 0$

$\cos x + \cos(x + 2x) = 0$

$\cos x + \cos x\cos 2x - \sin x\sin 2x = 0$

$\cos x + \cos x(2\cos^2 x - 1) - \sin x(2\sin x\cos x) = 0$

$\cos x + 2\cos^3 x - \cos x - 2\sin^2 x\cos x = 0$

$2\cos^3 x - 2(1 - \cos^2 x)\cos x = 0$

$2\cos^3 x - 2\cos x(1 - \cos^2 x) = 0$

$2\cos^3 x - 2\cos x + 2\cos^3 x = 0$

$2\cos x(\cos^2 x + \cos x - 1) = 0$

12. $\sin^2 x = 4 - 2\cos^2 x$

$1 - \cos^2 x = 4 - 2\cos^2 x$

$\cos^2 x + 1 - 4 = 0$

$\cos^2 x - 3 = 0$

$\cos^2 x = 3$

$\sqrt{\cos^2 x} = \pm\sqrt{3}$

$\sqrt{3} = 1.73$

$\cos x = \pm\sqrt{3}$

undefined, no solution