Key

4.23 Solving Trig Equations with Sum/Difference Identities Wksht Date: __________

Solve the equation for the variable on the interval $[0, 2\pi]$.

$* \sin 2x = 2\sin x\cos x$

1. $\sin x + \sqrt{2} = -\sin x$

$+\sin x \quad +\sin x$

$2\sin x + \sqrt{2} = 0$

$-\sqrt{2} \quad -\sqrt{2}$

$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

2. $\sin 2x \cos x = \sin x$

$2\sin x\cos x\cos x = \sin x$

$2\sin\cos^2 x - \sin x = 0$

$\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$2\cos^2 x - 1 = 0$

$\cos^2 x = 1/2$

$\cos x = \pm\sqrt{1/2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, 7\pi/4}$

3. $\cos(x - \pi) = \frac{\sqrt{2}}{2}$

$* \cos(a-b) = \cos a\cos b + \sin a\sin b$

$\cos x\cos\pi + \sin x\sin\pi = \frac{\sqrt{2}}{2}$

$\cos x(-1) + \sin x(0) = \sqrt{2}/2$

$-\cos x = \frac{\sqrt{2}}{2}$

$\cos x = -\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{3\pi}{4}, \frac{5\pi}{4}}$

4. $\sin(x - \pi) = -\frac{\sqrt{3}}{2}$

$* \sin(a-b) = \sin a\cos b - \cos a\sin b$

$\sin x\cos\pi - \cos x\sin\pi = -\frac{\sqrt{3}}{2}$

$\sin x(-1) - \cancel{\cos x(0)} = -\frac{\sqrt{3}}{2}$

$-\sin x = -\frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

5. $\sin\left(x + \frac{\pi}{4}\right) + 1 = \sin\left(\frac{\pi}{4} - x\right)$

$* \sin(a+b) = \sin a\cos b + \cos a\sin b$

$\sin x\cos\pi/4 + \cancel{\cos x\sin\pi/4} + 1 = \cancel{\sin\pi/4\cos x} - \cos\pi/4\sin x$

$+\cos\pi/4\sin x \qquad +\cos\pi/4\sin x$

$2\sin x\cos\pi/4 + 1 = 0$

$\cancel{2}\sin x\left(\frac{\sqrt{2}}{\cancel{2}}\right) = -1$

$\sqrt{2}\sin x = -1$

$\sin x = \frac{-1}{\sqrt{2}}$

$\sin x = \frac{-\sqrt{2}}{2}$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

6. $\tan\left(x + \frac{\pi}{4}\right) = 1$

$* \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$

$\frac{\tan x + \tan(\pi/4)}{1 - \tan x\tan\pi/4} = \frac{1}{1}$

$\tan x + \tan\pi/4 = 1(1 - \tan x\tan\pi/4)$

$\tan x + 1 = 1 - \tan x(1)$

$2\tan x = 0$

$\tan x = 0$

$\boxed{x = 0, \pi}$

$+2\pi n,\ n \in \mathbb{Z}$

Replace using $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

Solve the equation for all values of the variable.

7. $3\sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sqrt{\sec^2 x} = \pm\sqrt{\frac{4}{3}}$

$\sec x = \pm \frac{2}{\sqrt{3}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} + 2\pi n,\ n \in \mathbb{Z}}$

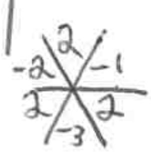
8. $2\sin^2 x + 3\cos x = 3$

$2(1-\cos^2 x) + 3\cos x = 3$

$2 - 2\cos^2 x + 3\cos x = 3$

$0 = 2\cos^2 x - 3\cos x + 1$

$*$ factor $2x^2 - 3x + 1$



$(x-1)(x-\frac{1}{2})$

$(x-1)(2x-1)$

$(\cos x - 1)(2\cos x - 1) = 0$

$\cos x - 1 = 0$ | $2\cos x - 1 = 0$

$\cos x = 1$ | $\cos x = \frac{1}{2}$

$\boxed{x = 0 \quad x = \frac{\pi}{3}, \frac{5\pi}{3} \quad + 2\pi n,\ n \in \mathbb{Z}}$

9. $\sin\left(\frac{\pi}{2} - x\right) = -1$

$*\ \sin(a-b) = \sin a \cos b - \cos a \sin b$

$\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x = -1$

$(1)\cos x - \cancel{(0)\sin x} = -1$

$\cos x = -1$

$\boxed{x = \pi \quad + 2\pi n,\ n \in \mathbb{Z}}$

10. $\cos\left(x + \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{6}\right) + 1$

$\cancel{\cos x \cos\frac{\pi}{6}} - \underline{\sin x \sin\frac{\pi}{6}} = \cancel{\cos x \cos\frac{\pi}{6}} + \underline{\sin x \sin\frac{\pi}{6}} + 1$

$-2\sin x \sin\frac{\pi}{6} = 1$

$-\cancel{2}\sin x\left(\frac{1}{\cancel{2}}\right) = 1$

$-\sin x = 1$

$\sin x = -1$

$\boxed{x = \frac{3\pi}{2} + 2\pi n,\ n \in \mathbb{Z}}$

11. $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2\left(\sin x \cos\pi + \cos x \sin\pi\right) = 0$

$+ 2\sin x \cos\pi + 2\cos x \sin\pi$

$\frac{\tan x + 0}{1 - \tan x (0)} + 2\sin x(-1) + \cancel{2\cos x(0)} = 0$

$\frac{\tan x}{1-0} - 2\sin x = 0$

$\tan x - 2\sin x = 0$

$\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x\left(\frac{1}{\cos x} - 2\right) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$\frac{1}{\cos x} - 2 = 0$

$\frac{1}{\cos x} = \frac{2}{1}$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

$\boxed{x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n,\ n \in \mathbb{Z}}$