Key

4.24 Solving Trig Equations with Functions of Multiple Angles Wksht **Date** ____________

Solve the equation for the variable on the interval $[0, 2\pi)$.

1. $\sin 3x = 1$ ← $\sin\theta = 1$

$3x = \sin^{-1}(1)$ Add $2\pi\left(+\frac{4\pi}{2}\right)$

$\frac{1}{3}\left(3x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}\right)\cdot\frac{1}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \cancel{\frac{13\pi}{6}}, \cancel{\frac{17\pi}{6}}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

2. $\cos 4x = 1$ $\cos\theta = 1$

$4x = \cos^{-1}(1)$

$\frac{1}{4}\left(4x = 0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi\right)\frac{1}{4}$

$\boxed{x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}}, \cancel{2\pi}, \cancel{\frac{5\pi}{2}}$

3. $\sec 2x = -2$ ← $\cos 2x = -\frac{1}{2}$

$2x = \sec^{-1}(-2)$

$2x = \cos^{-1}\left(-\frac{1}{2}\right)$ Add 2π or $\left(+\frac{6\pi}{3}\right)$

$\frac{1}{2}\left[2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}\right]\frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}, \cancel{\frac{7\pi}{3}}$

4. $\cos x + \sin x \tan x = 2$

$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right) = 2$

$\cos x + \frac{\sin^2 x}{\cos x} = 2$

$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = 2$

$\frac{\overbrace{\cos^2 x + \sin^2 x}^{1}}{\cos x} = 2$

$\frac{1}{\cos x} = 2$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

5. $\cos\left(x + \frac{\pi}{2}\right) = \tan\frac{\pi}{4}$

* $\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = \tan\left(\frac{\pi}{4}\right)$

$\cancel{\cos x(0)} - \sin x(1) = 1$

$-\sin x = 1$

$\sin x = -1$

$\boxed{x = \frac{3\pi}{2}}$

6. $2\sec^2 x + 2\tan^2 x = 14$ * $1 + \tan^2 x = \sec^2 x$

$2(1+\tan^2 x) + 2\tan^2 x = 14$

$2 + 2\tan^2 x + 2\tan^2 x = 14$

$2 + 4\tan^2 x = 14$

$-2 \qquad -2$

$\frac{4\tan^2 x}{4} = \frac{12}{4}$

$\tan^2 x = 3$

$\sqrt{\tan^2 x} = \pm\sqrt{3}$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

$+2\pi n, n \in \mathbb{Z}$

Solve the equation for all values of the variable.

7. $2\sin^2 2x = 1$

$\frac{2\sin^2 2x}{2} = \frac{1}{2}$

$\sqrt{\sin^2(2x)} = \pm\sqrt{\frac{1}{2}}$

$\sin(2x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

Add $\frac{8\pi}{4}$

$\frac{1}{2}\left[2x = \sin^{-1}\left(\pm\frac{\sqrt{2}}{2}\right) \rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}\right]\frac{1}{2}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, \cancel{\frac{17\pi}{8}} + 2\pi n, n \in \mathbb{Z}$

8. $-2\cos 2x = \sqrt{3}$

$\cos(2x) = -\frac{\sqrt{3}}{2}$

Add $\left(+\frac{12\pi}{6}\right)$

$\frac{1}{2}\left[2x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \rightarrow \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}\right]\frac{1}{2}$

$\boxed{x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \cancel{\frac{29\pi}{12}} + 2\pi n, n \in \mathbb{Z}}$

9. $3\tan^2 2x = 1$

$\tan^2(2x) = \frac{1}{3}$

$\sqrt{\tan^2(2x)} = \pm\sqrt{\frac{1}{3}}$

$\tan(2x) = \pm\frac{1}{\sqrt{3}}$

$2x = \tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$

Add $\frac{12\pi}{6}$

$\frac{1}{2}\left[2x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}\right]\frac{1}{2} \rightarrow$

$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}, \cancel{\frac{25\pi}{12}} + 2\pi n, n \in \mathbb{Z}}$

10. $\csc^2 x + 3\csc x = -2$

$\csc^2 x + 3\csc x + 2 = 0$

* factor $x^2 + 3x + 2 = 0$

$(x+2)(x+1)$

$(\csc x + 2)(\csc x + 1) = 0$

$\csc x + 2 = 0$	$\csc x + 1 = 0$
$\csc x = -2$	$\csc x = -1$
$\sin x = -\frac{1}{2}$	$\sin x = -1$
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$	$x = \frac{3\pi}{2}$

$\boxed{+2\pi n, n \in \mathbb{Z}}$

11. $\tan(x + \pi) + \cos\left(x + \frac{\pi}{2}\right) = 0$

$\frac{\tan x + \cancel{\tan \pi}}{1 - \cancel{\tan x \tan \pi}} + \cos x \cos(\pi/2) - \sin x \sin(\pi/2) = 0$

$\frac{\tan x}{1} + \cancel{\cos x (0)} - \sin x (1) = 0$

$\tan x - \sin x = 0$

$\frac{\sin x}{\cos x} - \sin x = 0$

$\sin x\left(\frac{1}{\cos x} - 1\right) = 0$

$\sin x = 0$ | $\frac{1}{\cos x} - 1 = 0$, $\frac{1}{\cos x} = 1$

$\cos x = 1$	$\sin x = 0$
$x = 0, \cancel{2\pi}$	$x = 0, \pi, \cancel{2\pi}$

$\boxed{x = 0, \pi + 2\pi n, n \in \mathbb{Z}}$

12. $\sin x - 1 = \cos x$

* Force trig identities to appear.

$(\sin x - 1)^2 = (\cos x)^2$

$(\sin x - 1)(\sin x - 1) = \cos^2 x$

* $\cos^2 x = 1 - \sin^2 x$

$\sin^2 x - 2\sin x + 1 = 1 - \sin^2 x$

$2\sin^2 x - 2\sin x = 0$

$2\sin x(\sin x - 1) = 0$

$2\sin x = 0$	$\sin x - 1 = 0$
$\sin x = 0$	$\sin x = 1$
$x = 0, \pi$	$x = \frac{\pi}{2}$

$\boxed{x = 0, \pi, \pi/2 + 2\pi n, n \in \mathbb{Z}}$