

Key

4.25 Solving Trig Equations Review

Date: _____

Solve each equation for the principal values.

1. $\sqrt{2} \sin x - 1 = 0$

$$\sqrt{2} \sin x = 1$$

$$\sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$X = \frac{\pi}{4}$

2. $2 \cos x + 1 = 0$

$$\cos x = -\frac{1}{2}$$

$X = \frac{2\pi}{3}$

3. $\sin 2x - 1 = 0$

$$\sin(2x) = 1$$

$$2x = \sin^{-1}(1)$$

4. $\cos^2 x = \cos x$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x - 1 = 0$$

$X = \frac{\pi}{2}$

$\cos x = 1$
 $X = 0$

Solve each equation for x if $0^\circ \leq x < 360^\circ$

5. $2 \cos^2 x + 3 \cos x - 2 = 0$

~~factor~~ $2x^2 + 3x - 2$

| | |
|---|------------------------|
| $\begin{array}{r} 4 \\ \times \\ -1 \\ \hline 2 \\ \times \\ 2 \\ \hline 2 \end{array}$ | $(x+2)(x-\frac{1}{2})$ |
| $\begin{array}{r} 2 \\ \times \\ 2 \\ \hline 4 \end{array}$ | $(x+2)(2x-1)$ |

$$(\cos x + 2)(2 \cos x - 1) = 0$$

$$\cos x = -2 \quad \cos x = \frac{1}{2}$$

undefined

$X = 60^\circ, 300^\circ$

6. $\cos x \tan x = \frac{1}{2}$

$$\cos x \left(\frac{\sin x}{\cos x} \right) = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$X = 30^\circ, 150^\circ$

7. $\sin x = 1 + \cos^2 x$

$\cos^2 x + \sin^2 x = 1$
 $\cos^2 x = 1 - \sin^2 x$

$$\sin x = 1 + (1 - \sin^2 x)$$

$$\sin x = 1 + 1 - \sin^2 x$$

$$\sin^2 x + \sin x - 2 = 0$$

* factor $x^2 + x - 2 = 0$

$$(x+2)(x-1) = 0$$

$$(\sin x + 2)(\sin x - 1) = 0$$

$$\sin x + 2 = 0 \quad \sin x - 1 = 0$$

$$\sin x = -2 \quad \sin x = 1$$

undefined

$X = 90^\circ$

Solve each equation for x if $0 \leq x < 2\pi$.

8. $\cot^2 x - \csc x = 1$

* $1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$$\csc^2 x - 1 - \csc x - 1 = 0$$

$$\csc^2 x - \csc x - 2 = 0$$

* factor $x^2 - x - 2 = 0$

$$(x-2)(x+1) = 0$$

$$(\csc x - 2)(\csc x + 1) = 0$$

$$\csc x - 2 = 0 \quad \csc x + 1 = 0$$

$$\csc x = 2 \quad \csc x = -1$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

$X = \frac{\pi}{6}, \frac{5\pi}{6}$

$X = \frac{3\pi}{2}$

9. $\sin x = \cos 2x - 1$

$$\sin x = (1 - 2\sin^2 x) - 1$$

$$\sin x = 1 - 2\sin^2 x - 1$$

$$2\sin^2 x + \sin x = 0$$

$$\sin x (2\sin x + 1) = 0$$

$$\sin x = 0 \quad 2\sin x + 1 = 0$$

$X = 0, \pi$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$X = \frac{7\pi}{6}, \frac{11\pi}{6}$

10. $\sin 2x = -\sin x$

$$2 \sin x \cos x + \sin x = 0$$

$\frac{2 \sin x \cos x}{\sin x} + \frac{\sin x}{\sin x} = 0$

$$\sin x (2 \cos x + 1) = 0$$

$$\sin x = 0 \quad 2 \cos x + 1 = 0$$

$X = 0, \pi$

$$\cos x = -\frac{1}{2}$$

$X = \frac{2\pi}{3}, \frac{4\pi}{3}$

11. Solve $\sin x + \cos x = 0$ $0 \leq x < \pi$.

$$\sin x = -\cos x$$

$$\tan x = -1$$

$$x = \frac{3\pi}{4}$$

$$\frac{\sin x}{\cos x} = \frac{-\cos x}{\cos x}$$

$$x = \cancel{\frac{\pi}{4}} \frac{3\pi}{4}$$

Solve each equation for all values of x .

12. $-1 - 3\sin x = \cos 2x$

$$= -1 - 2\sin^2 x$$

$$-1 - 3\sin x = -1 - 2\sin^2 x$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

$$\text{* factor } 2x^2 - 3x - 2 = 0$$

| | | | | |
|----|----|----|----------------|---------------|
| -4 | -4 | 1 | $(x-2)(x+1/2)$ | |
| 2 | 2 | -3 | | $(x-2)(2x+1)$ |
| 2 | 2 | -3 | | |

$$(\sin x - 2)(2\sin x + 1) = 0$$

$$\sin x = 2$$

$$\sin x = -1/2$$

undefined

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$+ 2\pi n, n \in \mathbb{Z}$$

14. $2\cos^2 x = 3\sin x$

$$2(1 - \sin^2 x) = 3\sin x$$

$$2 - 2\sin^2 x = 3\sin x$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

$$\text{* factor } 2x^2 + 3x - 2$$

| | | | | |
|---|----|----|----------------|---------------|
| 4 | -4 | -1 | $(x+2)(x-1/2)$ | |
| 2 | 3 | 2 | | $(x+2)(2x-1)$ |
| 2 | 3 | 2 | | |

$$(\sin x + 2)(2\sin x - 1) = 0$$

$$\sin x + 2 = 0$$

$$\sin x = 1/2$$

$$\sin x = -2$$

undefined

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$+ 2\pi n, n \in \mathbb{Z}$$

15. $3\tan^2 x = \sqrt{3}\tan x$

$$3\tan^2 x - \sqrt{3}\tan x = 0$$

$$\tan x (3\tan x - \sqrt{3}) = 0$$

$$\tan x = 0 \quad | \quad 3\tan x - \sqrt{3} = 0$$

$$x = 0, \pi \quad | \quad \tan x = \frac{\sqrt{3}}{3} \text{ or } \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6} + 2\pi n, n \in \mathbb{Z}$$