

4.26 Unit 4B Test Review WS #2 (Solving Trig Equations)

Key

Solve on the interval $[0, 2\pi)$

1) $2\cos^2 x + 5\sin x = -1$

$\sin^2 x + \cos^2 x = 1$
 $\cos^2 x = 1 - \sin^2 x$

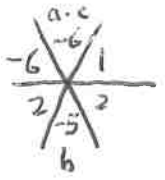
$2\cos^2 x + 5\sin x + 1 = 0$

$2(1 - \sin^2 x) + 5\sin x + 1 = 0$

$2 - 2\sin^2 x + 5\sin x + 1 = 0$

$0 = 2\sin^2 x - 5\sin x - 3$

* factor $2x^2 - 5x - 3$



$(x-3)(x+1/2)$
 $(x-3)(2x+1)$

$(\sin x - 3)(2\sin x + 1) = 0$

$\sin x = 3$

none

$\sin x = -1/2$
 $x = 7\pi/6, 11\pi/6$

2) $4\cos^2 x - 3 = 0$

$\cos^2 x = 3/4$

$\sqrt{\cos^2 x} = \pm\sqrt{3/4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3) $\cos(\frac{3\pi}{2} - x) = -\frac{1}{2}$

* $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(\frac{3\pi}{2}) \cos x + \sin(\frac{3\pi}{2}) \sin x = -1/2$

$(0) \cos x + (-1) \sin x = -1/2$

$-\sin x = -1/2$

$\sin x = 1/2$

$x = \pi/6, 5\pi/6$

4) $3\cos x = -2\sin^2 x$

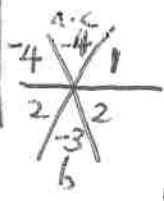
$\sin^2 x = 1 - \cos^2 x$

$3\cos x = -2(1 - \cos^2 x)$

$3\cos x = -2 + 2\cos^2 x$

$0 = 2\cos^2 x - 3\cos x - 2$

* factor $2x^2 - 3x - 2$



$(x-2)(x+1/2)$

$(x-2)(2x+1)$

$(\cos x - 2)(2\cos x + 1) = 0$

$\cos x - 2 = 0$

$\cos x = 2$

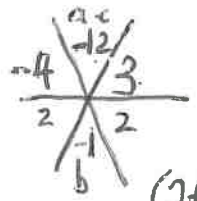
none

$\cos x = -1/2$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

5) $2\tan^2 x - \tan x - 6 = 0, 0 \leq x < 2\pi$

* factor $2x^2 - 1x - 6$



$(x+3/2)(x-2)$

$(2x+3)(x-2)$

$(2\tan x + 3)(\tan x - 2) = 0$

$\tan x = -3/2$

$\tan x = 2$

$x = \tan^{-1}(-3/2)$

$x = \tan^{-1}(2)$

6) $\tan^2 3x = 3$

$\sqrt{\tan^2(3x)} = \pm\sqrt{3}$

$\tan(3x) = \pm\sqrt{3}$

$3x = \tan^{-1}(\pm\sqrt{3})$

find where $\tan \theta = \pm\sqrt{3}$

add $+6\pi/3$

$\frac{1}{3} [3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}]$

$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}, \frac{19\pi}{9}$

7) Solve on the interval $[0, 2\pi)$ $\tan 4\theta = -1$

$$\tan(4\theta) = -1$$

$$4\theta = \tan^{-1}(-1)$$

$$\frac{1}{4} \left[4\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4} \right] \frac{1}{4}$$

find where
 $\tan \theta = -1$
Add $+8\pi/4$

$$\theta = \frac{3\pi}{16}, \frac{7\pi}{16}, \frac{11\pi}{16}, \frac{15\pi}{16}, \frac{19\pi}{16}$$

Solve the equations for all values of the variable $(+2\pi n, n \in \mathbb{Z})$

8) $\cos 2x - \cos x = 0$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

* factor $2x^2 - x - 1$

$$(2x+1)(x-1)$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -1/2 \quad | \quad \cos x = 1$$

$\cos x = -1/2$	$\cos x = 1$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$	$x = 0$
$+2\pi n, n \in \mathbb{Z}$	

9) $\sec 3x = 2 \tan \frac{7\pi}{4}$

$$\sec(3x) = 2(-1)$$

$$\sec(3x) = -2$$

$$\cos(3x) = -\frac{1}{2}$$

$$3x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\frac{1}{3} \left[3x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{20\pi}{3} \right] \frac{1}{3}$$

$$x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{20\pi}{9}$$

$+2\pi n, n \in \mathbb{Z}$

10) $\cos 2x = -\frac{1}{\sqrt{2}} \rightarrow -\frac{\sqrt{2}}{2}$

$$\cos(2x) = -\frac{\sqrt{2}}{2}$$

$$2x = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$\frac{1}{2} \left[2x = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4} \right]$$

$x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$
$+2\pi n, n \in \mathbb{Z}$

11) $\tan(\pi + x) + \tan(x + \pi) = 2$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan \pi + \tan x}{1 - \tan \pi \tan x} + \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} = 2$$

$$\frac{0 + \tan x}{1 - 0} + \frac{\tan x + 0}{1 - 0} = 2$$

$$2 \tan x = 2$$

$$\tan x = 1$$

$x = \pi/4, 5\pi/4$
$+2\pi n, n \in \mathbb{Z}$

12) $\frac{3\sec^2 \theta}{4} = 1$

$$3\sec^2 \theta = 4$$

$$\sec^2 \theta = \frac{4}{3}$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\sqrt{\cos^2 \theta} = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} + 2\pi n, n \in \mathbb{Z}$$