4.27 Unit 4B Test Review WS #3(Solving Trig Equations)

Key

Solve on the interval $[0, 2\pi)$

1) $4\sec x + 8 = 0, \quad 0 \le x < 2\pi$

$\sec x = -\frac{8}{4}$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

2) $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\sqrt{\cot^2 x} = \pm\sqrt{\frac{1}{3}}$

$\cot x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

3) $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2(\sin x\cos\pi + \cos x\sin\pi) = 0$

$\frac{\tan x + 0}{1 - \tan x(0)} + 2\sin x(-1) + 2\cos x(0) = 0$

$\tan x - 2\sin x = 0$

$\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x\left(\frac{1}{\cos x} - 2\right) = 0$

$\sin x = 0$ | $\frac{1}{\cos x} - 2 = 0$

$x = 0, \pi$ | $\frac{1}{\cos x} = \frac{2}{1}$

$2\cos x = 1$

$\cos x = 1/2$

$x = \pi/3, 5\pi/3$

4) $\csc^2 x = \csc x + 2$

$\csc^2 x - \csc x - 2 = 0$

* factor $x^2 - x - 2$

$(x-2)(x+1)$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$ | $\csc x + 1 = 0$

$\csc x = 2$ | $\csc x = -1$

$\sin x = 1/2$ | $\sin x = -1$

$x = \pi/6, 5\pi/6$ | $x = 3\pi/2$

5) $\sin^2 x - 5\cos x = 5$

$1 - \cos^2 x - 5\cos x = 5$

$0 = \cos^2 x + 5\cos x + 4$

* factor $x^2 + 5x + 4$

$(x+1)(x+4)$

$(\cos x + 1)(\cos x + 4) = 0$

$\cos x + 1 = 0$ | $\cos x + 4 = 0$

$\cos x = -1$ | $\cos x = -4$

$x = \pi$ | none

6) $\cot^2 3x = 3$

* on interval $[0, \pi)$

$\sqrt{\cot^2(3x)} = \pm\sqrt{3}$

$\cot(3x) = \pm\sqrt{3}$

$3x = \cot^{-1}(\pm\sqrt{3})$

$+\frac{12\pi}{6}$

$\frac{1}{3}$ [$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$]

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ | ~~$\frac{19\pi}{18}, \frac{23\pi}{18}$~~

7) Solve on the interval $[0, 2\pi)$ $\sec 3\theta = -1$

$\sec(3\theta) = -1$

$\cos(3\theta) = -1$

$3\theta = \cos^{-1}(-1)$

$[3\theta = \pi, 3\pi, 5\pi, 7\pi, 9\pi]\frac{1}{3}$

$\theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ ~~$\frac{7\pi}{3}, 3\pi$~~

Solve the equations for all values of the variable → $+2\pi n, n \in \mathbb{Z}$

8) $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ | $2\sin x - 1 = 0$

$x = \pi/2, 3\pi/2$ | $\sin x = 1/2$

$x = \pi/6, 5\pi/6$

$+2\pi n, n \in \mathbb{Z}$

9) $\sin 2x = 2\tan\frac{5\pi}{4}$ $\tan\left(\frac{5\pi}{4}\right) = 1$

$\sin(2x) = 2(1)$

$\sin(2x) = 1$

$2x = \sin^{-1}(1)$ $+2\pi$ (or $\frac{4\pi}{2}$)

$\frac{1}{2}[2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}]$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ ~~$\frac{9\pi}{4}, \frac{13\pi}{4}$~~

$+2\pi n, n \in \mathbb{Z}$

10) $2\cos^2 x - 3\cos x = -1$

$2\cos^2 x - 3\cos x + 1 = 0$

*factor $2x^2 - 3x + 1 = 0$

$(x-1)(x-1/2)$

$(x-1)(2x-1)$

$(\cos x - 1)(2\cos x - 1) = 0$

$\cos x = 1$ | $\cos x = 1/2$

$x = 0$ | $x = \pi/3, 5\pi/3$

$x = 0, \pi/3, 5\pi/3$

$+2\pi n, n \in \mathbb{Z}$

11) $\cos 2x - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

*factor $2x^2 - x - 1$

$(2x+1)(x-1)$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ | $\cos x - 1 = 0$

$\cos x = -1/2$ | $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ | $x = 0, 2\pi$

$+2\pi n, n \in \mathbb{Z}$

12) $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$

$\sin x\cos(\pi/2) + \cos x\sin(\pi/2) - [\cos x\cos(3\pi/2) - \sin x\sin(3\pi/2)]$

$\sin x(0) + \cos x(1) - \cos x(0) + \sin x(-1)$

$\cos x - \sin x = 0$

$\frac{\cos x}{\cos x} = \frac{\sin x}{\cos x}$

$1 = \tan x$

$\tan x = 1$

$x = \pi/4, 5\pi/4$

$+2\pi n, n \in \mathbb{Z}$