Ch. 4.2a Notes

I. Sigma Notation

$$\sum_{i=2}^{5} a_i = a_2 + a_3 + a_4 + a_5$$

$\boxed{\text{Ex.1}}$ $\sum_{i=2}^{4} i^2+1 =$

II. Summation Formulas *Memorize these*

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$

5) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

$\boxed{\text{Ex.2}}$ $\sum_{i=1}^{8} (3i^2+2)$

$\boxed{\text{Ex.3}}$ $\sum_{i=1}^{10} (i+2)^2$

$\boxed{\text{Ex.4}}$ $\sum_{K=1}^{n} \frac{1}{n}(K^2-1)$

4.2a Notes (continued)

III. Limit as n approaches infinity

* Think back about finding horizontal asymptotes.

$\boxed{\text{Ex. 5}}$ If $s(n) = \frac{1}{n^2}\left[\frac{n(n+1)}{2}\right]$ then find $\lim_{n\to\infty} s(n)$

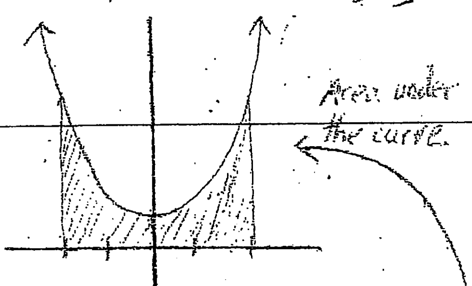
$\boxed{\text{Ex. 6}}$ Find $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$\boxed{\text{Ex. 7}}$ Find $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

4.2b - Riemann Sums

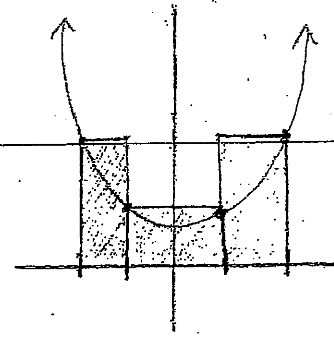
Riemann Sums - Using rectangles to estimate area of region.
(Area under a curve)

Consider the function
$f(x) = x^2 + 1 \quad [-2, 2]$



Suppose we want to estimate the area of the shaded region using a given number of rectangles.

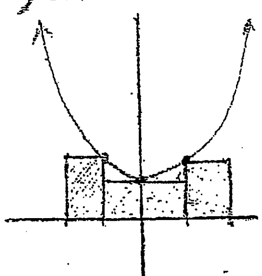
1) Upper rectangles or Circumscribed rectangles



* Using these rectangles will provide an overestimation of the area under the curve.

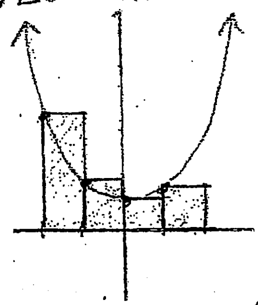
*Notice that one corner of each rectangle is on the graph. This ensures that the height of the rectangle is the same as the value of the function at the point where they connect.

2) Lower or Inscribed rectangles



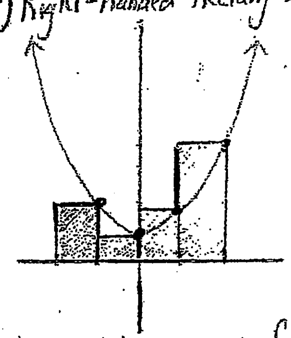
These rectangles provide an underestimation of the area under the curve

3) Left-handed Rectangles



The left corner of each rectangle attaches to the graph.

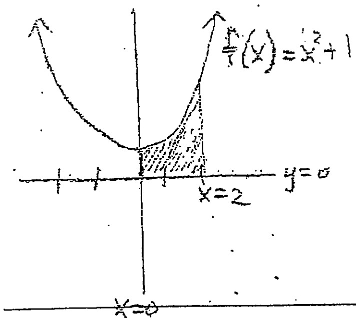
4) Right-handed Rectangles



The right corner of each rectangle attaches to the graph.

4.2b (continued)

$\boxed{\text{Ex. 1}}$ Use 4 rectangles to estimate upper (a) and lower (b) sums for the area bounded by $x=0$, $y=0$, $x=2$, and $y=x^2+1$



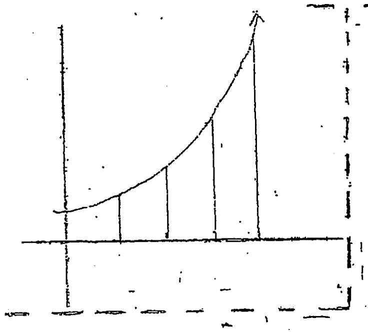
Step 1: Determine width of each rectangle.

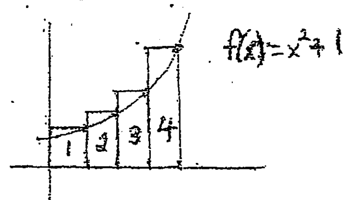
$$\boxed{\text{width} = \frac{b-a}{n}}$$

a = left endpoint
b = right endpoint
n = number of rectangles

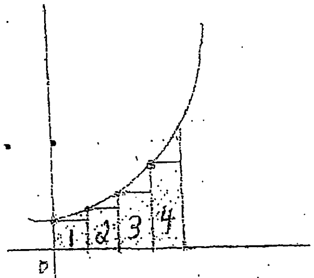
Step 2: Draw the graph. Section off each interval.

Step 3: Find sum of areas of appropriate rectangles.





b) Find lower sum

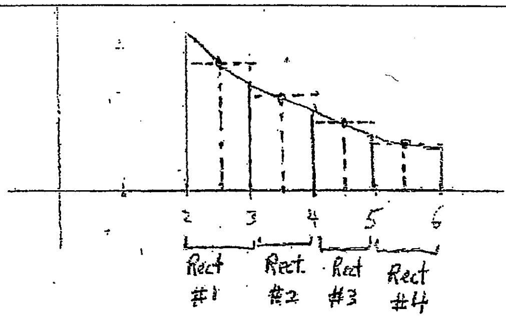


4.2b (continued)

Midpoint Rule: Similar to upper/lower sum but use the midpoint of each rectangle to calculate rectangle's height.

Ex. 2 Estimate area under curve $f(x)=\frac{8}{x^2+1}$ from $[2,6]$
Use midpoint rule with 4 subintervals.

$$\text{width}=\frac{b-a}{n}=\frac{6-2}{4}=\frac{4}{4}=1$$

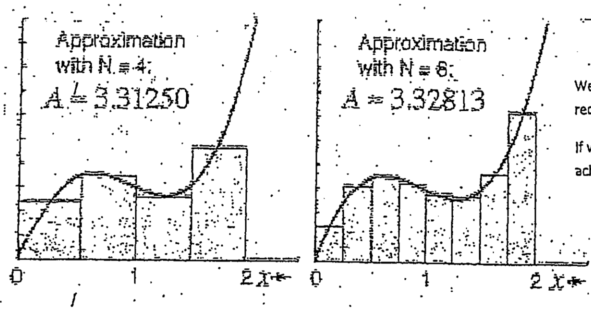


- Why would midpoint sum be a better approximation of area than upper or lower sum?

  This is because each rectangle has portions above and below the graph.

* Note: Midpoint sum is not the average between upper and lower sum!

4.2c Finding Exact Area using limits



We can continually improve the Area Approximation under the curve by increasing the number of rectangles : above (n = 4) and n = 8, _n = 16_

If we let n go out to infinity, (using limits) we will have something better than an approximation, we will achieve the actual area under the curve.

$$\text{Exact Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{b-a}{n}\right) \cdot f\left(a + \left(\frac{b-a}{n}\right) i\right) \right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (\text{width}) \cdot f(\underline{\text{left endpoint}} + \underline{\text{width} \cdot i})$$

Memorize → "width f left plus width times i"

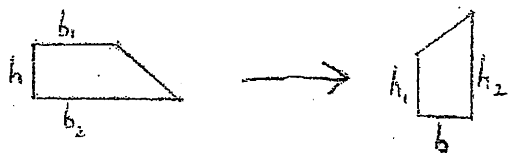
$\boxed{\text{Ex. 1}}$ Find exact area between $f(x) = 4 - x^2$ and x-axis from $[-2, 2]$

4.6 Trapezoids

*Better approximation than inscribed, circumscribed, or midpoint rectangles.

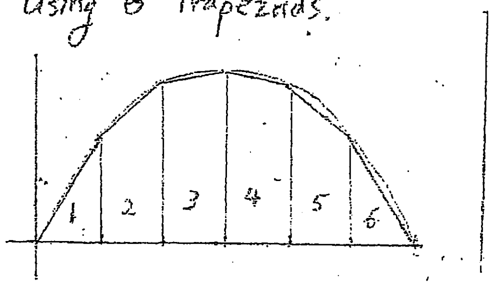
Trapezoidal Rule: Approximate Area of region using areas of trapezoids.

Review: Area of Trapezoid $= \frac{1}{2}h(b_1+b_2)$ or $\frac{h}{2}(b_1+b_2)$



$\boxed{\text{Area} = \frac{b}{2}(h_1+h_2)}$ or $\frac{1}{2}w(h_1+h_2)$

$\boxed{\text{Ex. 1}}$ Estimate area bounded by $f(x)=6x-x^2$ and the x-axis using 6 trapezoids.



Review 4.2 4.6 Formulas and Definitions:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 =$ $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i =$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 =$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 =$ $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) Area of Trapezoid: ______________________________ $Area = \frac{w}{2}(h_1 + h_2)$

6) Width formula: ________________________________ $width = \frac{b-a}{n}$

7) Limit Definition of Area under Curve $\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(a + width * i)$

Non-AP Calculus 4.2, 4.6 Formulas

Summation Formulas:

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

Riemann Sum: Estimate Area under curve using rectangles

a) Right-handed sum
b) Left-handed sum
c) Mid-Point Rule

Trapezoid Rule: ½(width)(h_1 +2 h_2+ 2 h_3 + ... h_n)

Area of Trapezoid: $Area = \frac{w}{2}[h_1 + h_2]$

Area of Rectangle: Area = (width) x (height)

Width of Intervals: $width = \frac{b-a}{n}$