

1) Find the Sum  $\sum_{k=3}^5 \frac{1 - k^2}{2k + 1}$

2) Use Sigma Notation to write the sum  $\frac{4}{\sqrt[7]{8-3}} + \frac{16}{\sqrt[7]{8-9}} + \frac{64}{\sqrt[7]{8-27}} + \dots + \frac{1024}{\sqrt[7]{8-243}}$

3) Use properties of summation to evaluate the sum:

$$\sum_{i=1}^{27} 2(i - 1)^2 + 3$$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis:  $f(x) = 2 + 5x^2$  in interval  $[4, 11]$  using 3 rectangles

a) Left endpoint (LRAM):

b) Right Endpoint(RRAM):

c) Middle rectangle (MRAM):

5) Selected values of a function,  $f$ , are given in the table below:

|      |   |   |   |   |    |    |    |    |    |
|------|---|---|---|---|----|----|----|----|----|
| x    | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 27 |
| f(x) | 6 | 1 | 2 | 4 | 6  | 7  | 12 | 7  | 10 |

- a) Use middle rectangles to approximate the area with 2 subintervals for  $f$  on the interval interval [2, 27]

|      |   |   |   |   |    |    |    |    |    |
|------|---|---|---|---|----|----|----|----|----|
| x    | 1 | 3 | 7 | 9 | 16 | 17 | 21 | 23 | 27 |
| f(x) | 6 | 5 | 2 | 4 | 6  | 7  | 12 | 7  | 10 |

- b) Use right-handed rectangles to approximate the area with 4 subintervals for  $f$  on the interval [7,21]

|      |   |   |   |   |    |    |    |    |    |
|------|---|---|---|---|----|----|----|----|----|
| x    | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 27 |
| f(x) | 6 | 1 | 2 | 4 | 6  | 7  | 12 | 7  | 10 |

- c) Use left-handed rectangles to approximate the area with 3 subintervals for  $f$  on the interval [4,23]

|      |   |   |   |   |    |    |    |    |    |
|------|---|---|---|---|----|----|----|----|----|
| x    | 2 | 4 | 7 | 9 | 16 | 17 | 21 | 23 | 30 |
| f(x) | 6 | 1 | 2 | 4 | 5  | 7  | 12 | 7  | 10 |

- d) Use trapezoids to approximate the area with 3 subintervals for  $f$  on the interval [7, 30]

Key

1) Find the Sum  $\sum_{k=3}^5 \frac{1-k^2}{2k+1} = \frac{1-3^2}{2(3)+1} + \frac{1-4^2}{2(4)+1} + \frac{1-5^2}{2(5)+1}$

$$= \frac{-8}{7} + \frac{-15}{9} + \frac{-24}{11} = \frac{-1153}{231} \approx \boxed{-4.991}$$

2) Use Sigma Notation to write the sum  $\frac{4}{\sqrt[7]{8-3}} + \frac{16}{\sqrt[7]{8-9}} + \frac{64}{\sqrt[7]{8-27}} + \dots + \frac{1024}{\sqrt[7]{8-243}}$

$$\sum_{i=1}^5 \frac{4^i}{\sqrt[7]{8-3^i}}$$

$i=1 \quad i=2 \quad i=3 \quad i=?$

3) Use properties of summation to evaluate the sum:

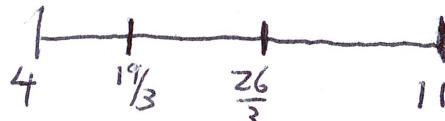
$$\sum_{i=1}^{27} 2(i-1)^2 + 3 = 2(i-1)(i-1) + 3 \rightarrow 2(i^2 - 2i + 1) + 3 \rightarrow 2i^2 - 4i + 2 + 3$$

$$\sum 2i^2 - 4i + 5$$

$$\left| \begin{array}{l} 2 \left[ \sum_{i=1}^{27} i^2 \right] - 4 \left[ \sum_{i=1}^{27} i \right] + 5 \left[ \sum_{i=1}^{27} 1 \right] \\ 2 \left[ \frac{n(n+1)(2n+1)}{6} \right] - 4 \left[ \frac{n(n+1)}{2} \right] + 5[n] \\ 2 \left[ \frac{27(28)(55)}{6} \right] - 4 \left[ \frac{27(28)}{2} \right] + 5[27] \end{array} \right| = 13860 - 1512 + 135 = \boxed{12483}$$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis:  $f(x) = 2 + 5x^2$  in interval  $[4, 11]$  using 3 rectangles

$$W = \frac{b-a}{n} = \frac{11-4}{3} = \frac{7}{3}$$



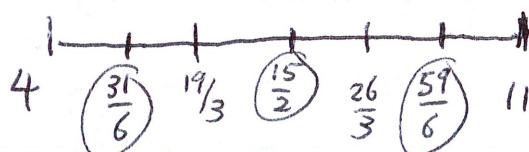
a) Left endpoint (LRAM):

$$\begin{aligned} \text{Area} &\approx \frac{7}{3} \left[ f(4) + f\left(\frac{19}{3}\right) + f\left(\frac{26}{3}\right) \right] \\ &= \frac{7}{3} [82 + 202.55 + 377.55] \end{aligned} \quad \boxed{1544.9}$$

b) Right Endpoint(RRAM):

$$\text{Area} = \frac{7}{3} \left[ f\left(\frac{19}{3}\right) + f\left(\frac{26}{3}\right) + f(11) \right] = \boxed{2769.926}$$

c) Middle rectangle (MRAM):



$$\begin{aligned} A &= \frac{7}{3} \left[ f\left(\frac{31}{6}\right) + f\left(\frac{15}{2}\right) + f\left(\frac{59}{6}\right) \right] \\ &= \boxed{2109.787} \end{aligned}$$

- 5) Selected values of a function,  $f$ , are given in the table below:

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11

|      |   |   |     |   |    |    |      |    |    |
|------|---|---|-----|---|----|----|------|----|----|
| x    | 2 | 4 | 7   | 9 | 16 | 17 | 21   | 23 | 27 |
| f(x) | 6 | 1 | (2) | 4 | 6  | 7  | (12) | 7  | 10 |

$$\text{Area} = \text{width} \times \text{height}$$

- a) Use middle rectangles to approximate the area with 2 subintervals for  $f$  on the interval interval [2, 27]

$$A = 14 \cdot f(7) + 11 \cdot f(21) \quad \left| A = 28 + 132 = \boxed{160} \right.$$

$$= 14(2) + 11(12)$$

|      |   |   |   |     |     |     |      |    |    |
|------|---|---|---|-----|-----|-----|------|----|----|
| x    | 1 | 3 | 7 | 9   | 16  | 17  | 21   | 23 | 27 |
| f(x) | 6 | 5 | 2 | (4) | (6) | (7) | (12) | 7  | 10 |

- b) Use right-handed rectangles to approximate the area with 4 subintervals for  $f$  on the interval [7, 21]

$$A = 2 \cdot f(9) + 7 \cdot f(16) + 1 \cdot f(17) + 4 \cdot f(21) \quad \left| = \boxed{105} \right.$$

$$= 2(4) + 7(6) + 1(7) + 4(12)$$

|      |   |     |   |     |    |     |    |    |    |
|------|---|-----|---|-----|----|-----|----|----|----|
| x    | 2 | 4   | 7 | 9   | 16 | 17  | 21 | 23 | 27 |
| f(x) | 6 | (1) | 2 | (4) | 6  | (7) | 12 | 7  | 10 |

- c) Use left-handed rectangles to approximate the area with 3 subintervals for  $f$  on the interval [4, 23]

$$A = 5 \cdot f(4) + 8 \cdot f(9) + 6 \cdot f(17) \quad \left| = \boxed{79} \right.$$

$$5(1) + 8(4) + 6(7)$$

|      |   |   |     |   |     |    |      |    |      |
|------|---|---|-----|---|-----|----|------|----|------|
| x    | 2 | 4 | 7   | 9 | 16  | 17 | 21   | 23 | 30   |
| f(x) | 6 | 1 | (2) | 4 | (5) | 7  | (12) | 7  | (10) |

$$\text{Area} = \frac{w}{2} [h_1 + h_2]$$

- d) Use trapezoids to approximate the area with 3 subintervals for  $f$  on the interval [7, 30]

$$A = \frac{9}{2} [f(7) + f(16)] + \frac{5}{2} [f(16) + f(21)] + \frac{9}{2} [f(21) + f(30)]$$

$$\frac{9}{2} [2 + 5] + \frac{5}{2} [5 + 12] + \frac{9}{2} [12 + 10] = \boxed{173}$$

$$31.5 + 42.5 + 99$$