4.2/4.6 Riemann Sums Classwork/HW

Finding a Sum In Exercises 1–6, find the sum. Use the summation capabilities of a graphing utility to verify your result.

1. $\sum_{i=1}^{6}(3i+2)$

2. $\sum_{k=3}^{9}(k^2+1)$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$

4. $\sum_{j=4}^{6}\frac{3}{j}$

Using Sigma Notation In Exercises 7–12, use sigma notation to write the sum.

7. $\frac{1}{5(1)}+\frac{1}{5(2)}+\frac{1}{5(3)}+\cdots+\frac{1}{5(11)}$

9. $\left[7\left(\frac{1}{6}\right)+5\right]+\left[7\left(\frac{2}{6}\right)+5\right]+\cdots+\left[7\left(\frac{6}{6}\right)+5\right]$

Evaluating a Sum In Exercises 13–20, use the properties of summation and Theorem 4.2 to evaluate the sum. Use the summation capabilities of a graphing utility to verify your result.

13. $\sum_{i=1}^{12} 7$

14. $\sum_{i=1}^{30} -18$

15. $\sum_{i=1}^{24} 4i$

16. $\sum_{i=1}^{16} (5i - 4)$

17. $\sum_{i=1}^{20} (i - 1)^2$

18. $\sum_{i=1}^{10} (i^2 - 1)$

Approximating the Area of a Plane Region In Exercises 25–30, use left and right endpoints and the given number of rectangles to find two approximations of the area of the region between the graph of the function and the x-axis over the given interval.

25. $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles (LRAM, RRAM)

26. $f(x) = 9 - x$, $[2, 4]$, 6 rectangles (RRAM)

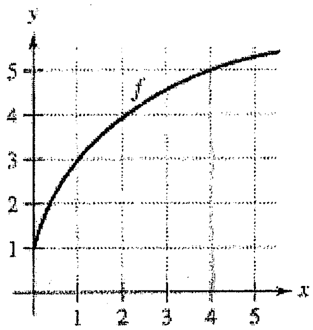
27. $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles (LRAM)

28. $g(x) = x^2 + 1$, $[1, 3]$, 8 rectangles (RRAM)

Using Upper and Lower Sums In Exercises 31 and 32, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

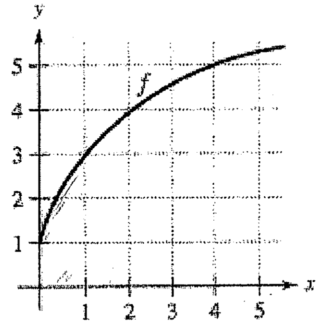
Upper Sum [0, 4]

31.



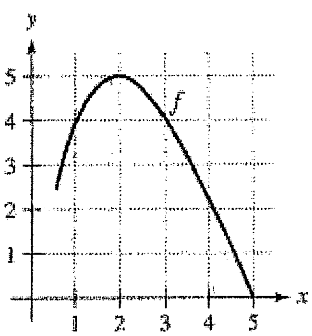
Lower Sum [0, 4]

31.



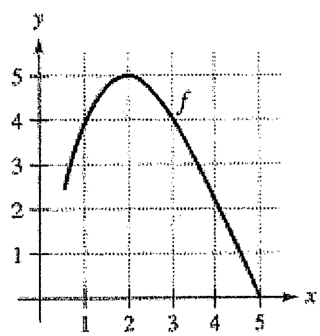
Upper Sum [1, 5]

32.



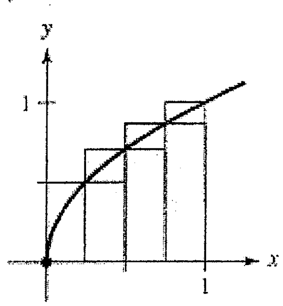
Lower Sum [1, 5]

32.



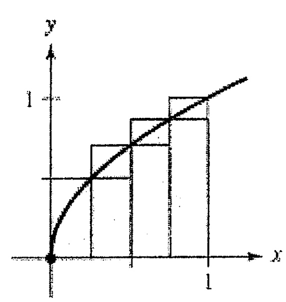
Upper Sum

33. $y = \sqrt{x}$



Lower Sum

33. $y = \sqrt{x}$



Ch. 4.6
Trapezoid Rule

Using the Trapezoidal Rule In Exercises 1–10, use the Trapezoidal Rule to approximate the value of the definite integral for the given value of n. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

1. $\int_0^2 x^2\,dx, \quad n = 4$

3. $\int_0^2 x^3\,dx, \quad n = 4$

5. $\int_1^3 x^3\,dx, \quad n = 6$

7. $\int_4^9 \sqrt{x}\,dx, \quad n = 8$

4.2/4.6 Riemann Sums Classwork/HW

Key

Finding a Sum In Exercises 1–6, find the sum. Use the summation capabilities of a graphing utility to verify your result.

1. $\sum_{i=1}^{6}(3i+2)$

$3\sum i + 2\sum 1$

$3\left(\frac{n(n+1)}{2}\right) + 2(n)$

$3\left(\frac{6(7)}{2}\right) + 2(6) = 63 + 12 = \boxed{75}$

2. $\sum_{k=3}^{9}(k^2+1)$

$3^2+1+4^2+1+5^2+1+6^2+1+7^2+1+8^2+1+9^2+1$

$= \boxed{287}$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$

$\frac{1}{0+1} + \frac{1}{1+1} + \frac{1}{2^2+1} + \frac{1}{3^2+1} + \frac{1}{4^2+1}$

$1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} = \boxed{\frac{158}{85}}$

4. $\sum_{j=4}^{6}\frac{3}{j}$

$\frac{3}{4} + \frac{3}{5} + \frac{3}{6} = \boxed{\frac{37}{20}}$

Using Sigma Notation In Exercises 7–12, use sigma notation to write the sum.

7. $\frac{1}{5(1)} + \frac{1}{5(2)} + \frac{1}{5(3)} + \cdots + \frac{1}{5(11)}$

$\sum_{i=1}^{11}\frac{1}{5(i)} = \boxed{\sum_{i=1}^{11}\frac{1}{5i}}$

9. $\left[7\left(\frac{1}{6}\right)+5\right] + \left[7\left(\frac{2}{6}\right)+5\right] + \cdots + \left[7\left(\frac{6}{6}\right)+5\right]$

$\sum_{i=1}^{6}\left[7\left(\frac{i}{6}\right)+5\right]$

Evaluating a Sum In Exercises 13–20, use the properties of summation and Theorem 4.2 to evaluate the sum. Use the summation capabilities of a graphing utility to verify your result.

13. $\sum_{i=1}^{12} 7$ $\quad 7\boxed{\sum 1} = 7n$

$7(12) = \boxed{84}$

14. $\sum_{i=1}^{30} -18$ $\quad = -18\boxed{\sum 1}$

$= -18n = -18(30)$

$= \boxed{-540}$

15. $\sum_{i=1}^{24} 4i$ $\quad = 4\boxed{\sum i}$

$4\left(\frac{n(n+1)}{2}\right) = 4\left(\frac{24(25)}{2}\right)$

$= \boxed{1200}$

16. $\sum_{i=1}^{16} (5i - 4)$ $\quad = 5\boxed{\sum i} - 4\boxed{\sum 1}$

$5\left(\frac{n(n+1)}{2}\right) - 4(n)$

$5\left[\frac{16(17)}{2}\right] - 4[16] = \boxed{616}$

17. $\sum_{i=1}^{20} (i - 1)^2$

$\sum (i-1)(i-1) = \sum (i^2 - 2i + 1)$

$\sum i^2 - \sum 2i + \sum 1$

$\sum i^2 - 2\sum i + \sum 1$

$\frac{n(n+1)(2n+1)}{6} - 2\left(\frac{n(n+1)}{2}\right) + n$

$= \frac{(20)(21)(41)}{6} - 2\left(\frac{20(21)}{2}\right) + 20 = 2870 - 420 = \boxed{2470}$

18. $\sum_{i=1}^{10} (i^2 - 1)$

$\sum i^2 - \sum 1$

$\frac{n(n+1)(2n+1)}{6} - n$

$\frac{(10)(11)(21)}{6} - 10 = \boxed{375}$

Approximating the Area of a Plane Region In Exercises 25–30, use left and right endpoints and the given number of rectangles to find two approximations of the area of the region between the graph of the function and the x-axis over the given interval.

25. $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles

$W = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2} = 0.5$

0 0.5 1 1.5 2

$\text{Area}(LRAM) = (0.5)(f(0)) + 0.5(f(0.5)) + 0.5 \cdot f(1) + 0.5 \cdot f(1.5) = \frac{1}{2}(5+6+7+8) = \boxed{13}$

$\text{Area}(RRAM) = 0.5 \cdot f(0.5) + 0.5 \cdot f(1) + 0.5 \cdot f(1.5) + 0.5 \cdot f(2) = \frac{1}{2}(6+7+8+9) = \boxed{15}$

26. $f(x) = 9 - x$, $[2, 4]$, 6 rectangles

$W = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}$

2 7/3 8/3 9/3 10/3 11/3 12/3

$\text{Area}(RRAM) = \frac{1}{3}\left[f(7/3) + f(8/3) + f(3) + f(10/3) + f(11/3) + f(4)\right]$

$= \frac{1}{3}\left[\frac{20}{3} + \frac{19}{3} + 6 + \frac{17}{3} + \frac{16}{3} + \frac{15}{3}\right] = \frac{35}{3} \approx \boxed{11.667}$

27. $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles

$W = \frac{5-2}{6} = \frac{3}{6} = 1/2$

2 5/2 6/2 7/2 8/2 9/2 5

$\text{Area}(LRAM) = \frac{1}{2}\left[f(2) + f(5/2) + f(3) + f(7/2) + f(4) + f(9/2)\right]$

$= \frac{1}{2}\left[5 + 9 + 14 + 20 + 27 + 35\right] = \boxed{55}$

28. $g(x) = x^2 + 1$, $[1, 3]$, 8 rectangles

$W = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$

1 5/4 6/4 7/4 2 9/4 10/4 11/4 3

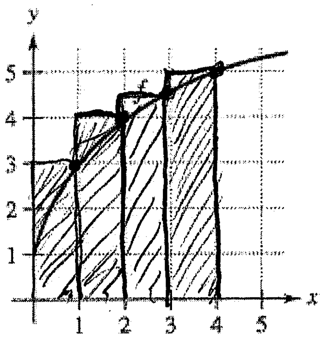
$\text{Area}(RRAM) = \frac{1}{4}\left[f(5/4) + f(6/4) + f(7/4) + f(2) + f(9/4) + f(10/4) + f(11/4) + f(3)\right]$

$= \frac{1}{4}\left[\frac{41}{6} + \frac{13}{4} + \frac{65}{16} + 5 + \frac{97}{16} + \frac{29}{4} + \frac{137}{16} + 10\right] = \boxed{11.6875}$

Using Upper and Lower Sums In Exercises 31 and 32, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

Upper Sum $[0,4]$

31.

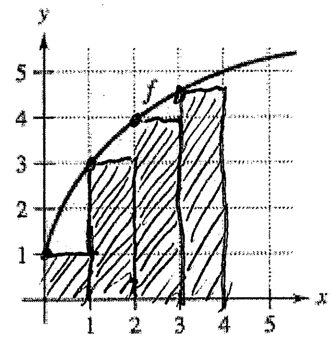


$1(3)+1(4)+1(4.5)+1(5)$

$3+4+4.5+5=\boxed{16.5}$

Lower Sum $[0,4]$

31.

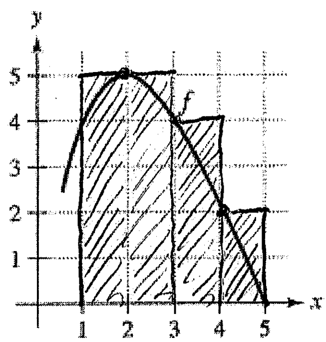


$1(1)+1(3)+1(4)+1(4.5)$

$=\boxed{12.5}$

Upper Sum $[1,5]$

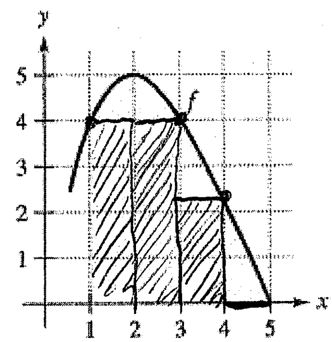
32.



$1(5)+1(5)+1(4)+1(2)=\boxed{16}$

Lower Sum $[1,5]$

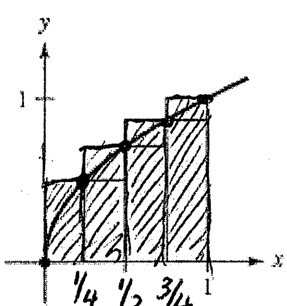
32.



$1(4)+1(4)+1(2)+1(0)=\boxed{10}$

Upper Sum

33. $y = \sqrt{x}$

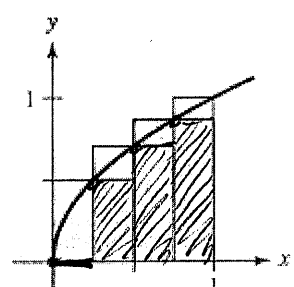


$W=\frac{1-0}{4}=1/4$

$S(4)=\frac{1}{4}\cdot\left[y(1/4)+y(1/2)+y(3/4)+y(1)\right]\approx\boxed{0.768}$

Lower Sum

33. $y = \sqrt{x}$



$w=1/4$

$s(4)\approx\frac{1}{4}\left[y(0)+y(1/4)+y(1/2)+y(3/4)\right]$

$=\boxed{0.518}$

Using the Trapezoidal Rule In Exercises 1–10, use the Trapezoidal Rule to approximate the value of the definite integral for the given value of n. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

Ch. 4.6
Trapezoid Rule

$A = \frac{w}{2}[h_1 + 2h_2 + 2h_3 + \dots + h_n]$

$A = w \cdot \frac{1}{2}[h_1 + 2h_2 + 2h_3 + \dots + h_n]$

1. $\int_0^2 x^2\,dx, \quad n = 4$

$w = \frac{2-0}{4} = \frac{1}{2}$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

$\text{Area} \approx \frac{0.5}{2}[f(0) + 2f(\frac{1}{2}) + 2f(1) + 2f(\frac{3}{2}) + f(2)]$

$= \boxed{2.75}$

3. $\int_0^2 x^3\,dx, \quad n = 4$

$w = \frac{2-0}{4} = \frac{1}{2}$

$\text{Area} \approx \frac{0.5}{2}[f(0) + 2f(\frac{1}{2}) + 2f(1) + 2f(\frac{3}{2}) + f(2)]$

$= \boxed{4.25}$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

5. $\int_1^3 x^3\,dx, \quad n = 6$

$w = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$

$\text{Area} \approx \frac{1}{3} \cdot \frac{1}{2}[f(1) + 2f(\frac{4}{3}) + 2f(\frac{5}{3}) + 2f(2) + 2f(\frac{7}{3}) + 2f(\frac{8}{3}) + f(3)]$

$\boxed{20.22}$

$1 \quad \frac{4}{3} \quad \frac{5}{3} \quad 2 \quad \frac{7}{3} \quad \frac{8}{3} \quad 3$

7. $\int_4^9 \sqrt{x}\,dx, \quad n = 8$

$w = \frac{9-4}{8} = \frac{5}{8}$

$\text{Area} = \frac{1}{2} \cdot \frac{5}{8}[f(4) + 2f(\frac{37}{8}) + 2f(\frac{42}{8}) + 2f(\frac{47}{8}) + 2f(\frac{52}{8}) + 2f(\frac{57}{8}) + 2f(\frac{62}{8}) + 2f(\frac{67}{8}) + f(\frac{72}{8})]$

$= \boxed{12.664}$

$4 \quad \frac{37}{8} \quad \frac{42}{8} \quad \frac{47}{8} \quad \frac{52}{8} \quad \frac{57}{8} \quad \frac{62}{8} \quad \frac{67}{8} \quad \frac{72}{8} = 9$