Non-AP Calculus **<u>4-2, 4-6 Quiz Review Worksheet #3</u>**

1) *Find the Sum* $\displaystyle\sum_{k=4}^{7} \frac{k}{3^k - k^2}$

2) Use Sigma Notation to write the sum $\dfrac{4}{5-1} + \dfrac{5}{5-8} + \dfrac{6}{5-27} + \cdots \dfrac{9}{5-216}$

3) Use properties of summation to evaluate the sum:

$$\sum_{i=1}^{31} 1 - 3(i-1)^2$$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis: $f(x) = 2 + 5x^2 \quad in\ interval\ [1, 8] \quad using\ 3\ rectangles$

a) Left endpoint (LRAM):

b) Right Endpoint(RRAM):

c) Middle rectangle (MRAM):

5) Selected values of a function, *f*, are given in the table below:

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

a) Use middle rectangles to approximate the area with 3 subintervals for *f* on the interval interval [4, 23]

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

b) Use right-handed rectangles to approximate the area with 4 subintervals for *f* on the interval [2, 25]

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

c) Use left-handed rectangles to approximate the area with 2 subintervals for *f* on the interval [7, 25]

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	5	7	12	7	10

d) Use trapezoids to approximate the area with 2 subintervals for *f* on the interval [2, 25]

Non-AP Calculus **4-2, 4-6 Quiz Review Worksheet #3** Key

1) *Find the Sum* $\sum_{k=4}^{7} \frac{k}{3^k - k^2}$

$$\frac{4}{3^4-4^2} + \frac{5}{3^5-5^2} + \frac{6}{3^6-6^2} + \frac{7}{3^7-7^2} = \frac{4}{65} + \frac{5}{218} + \frac{2}{231} + \frac{7}{2138} \approx \boxed{0.096}$$

2) Use Sigma Notation to write the sum $\frac{4}{5-1} + \frac{5}{5-8} + \frac{6}{5-27} + \cdots \frac{9}{5-216}$

$i=1 \quad i=2 \quad i=3 \quad i=?$

$$\boxed{\sum_{i=1}^{6} \frac{i+3}{5-i^3}}$$

3) Use properties of summation to evaluate the sum:

$$\sum_{i=1}^{31} 1 - 3(i-1)^2 \rightarrow 1-3(i-1)(i-1) \rightarrow 1-3(i^2-2i+1) \rightarrow 1-3i^2+6i-3 \rightarrow \sum -3i^2+6i-2$$

$$\sum -3i^2 + \sum 6i - \sum 2$$

$$-3\boxed{\sum i^2} + 6\boxed{\sum i} - 2\boxed{\sum 1}$$

$$-3\left[\frac{n(n+1)(2n+1)}{6}\right] + 6\left[\frac{n(n+1)}{2}\right] - 2[n]$$

$$-3\left[\frac{31(32)(63)}{6}\right] + 6\left[\frac{31(32)}{2}\right] - 2[31]$$

-31248
$+2976$
-62
$\boxed{-28,334}$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis: $f(x) = 2 + 5x^2$ *in interval* $[1, 8]$ *using* 3 *rectangles*

$$W = \frac{b-a}{n} = \frac{8-1}{3} = \frac{7}{3}$$

$1 \quad \frac{10}{3} \quad \frac{17}{3} \quad 8$

a) Left endpoint (LRAM):

$$A = \frac{7}{3}\left[f(1) + f\left(\frac{10}{3}\right) + f\left(\frac{17}{3}\right)\right]$$

$$A = \frac{7}{3}\left[7 + \frac{518}{9} + \frac{1463}{9}\right] = \boxed{\frac{14308}{27} \approx 529.926}$$

b) Right Endpoint(RRAM):

$$A = \frac{7}{3}\left[f\left(\frac{10}{3}\right) + f\left(\frac{17}{3}\right) + f(8)\right]$$

$$\frac{7}{3}\left[\frac{518}{9} + \frac{1463}{9} + 322\right] \approx \boxed{1264.926}$$

c) Middle rectangle (MRAM):

$1 \quad \frac{13}{6} \quad \frac{10}{3} \quad \frac{9}{2} \quad \frac{17}{3} \quad \frac{41}{6} \quad 8$

$$A \approx \frac{7}{3}\left[f\left(\frac{13}{6}\right) + f\left(\frac{9}{2}\right) + f\left(\frac{41}{6}\right)\right]$$

$$\approx \boxed{849.787 = \frac{91777}{108}}$$

5) Selected values of a function, *f*, are given in the table below:

5 8 6

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

Area = width × height

a) Use middle rectangles to approximate the area with 3 subintervals for *f* on the interval interval [4, 23]

$$\text{Area} \approx 5\cdot f(7) + 8\cdot f(16) + 6\cdot f(21)$$
$$= 5(2) + 8(6) + 6(12) = \boxed{130}$$

5 9 5 4

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

b) Use right-handed rectangles to approximate the area with 4 subintervals for *f* on the interval [2, 25]

$$\text{Area} \approx 5\cdot f(7) + 9\cdot f(16) + 5\cdot f(21) + 4\cdot f(25)$$
$$= 5(2) + 9(6) + 5(12) + 4(10) = \boxed{164}$$

10 8

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	6	7	12	7	10

c) Use left-handed rectangles to approximate the area with 2 subintervals for *f* on the interval [7, 25]

$$\text{Area} \approx 10\cdot f(7) + 8\cdot f(17)$$
$$10(2) + 8(7) = \boxed{76}$$

14 9

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	2	4	5	7	12	7	10

$\text{Area} = \frac{W}{2}[h_1 + h_2]$

d) Use trapezoids to approximate the area with 2 subintervals for *f* on the interval [2, 25]

$$\text{Area} \approx \frac{14}{2}[f(2)+f(16)] + \frac{9}{2}[f(16)+f(25)]$$
$$= 7[6+5] + \frac{9}{2}[5+10] = \boxed{144.5}$$
$$7(11) + 4.5(15)$$