

1) Find the Sum $\sum_{k=4}^7 \frac{k}{3^k - k^2}$

2) Use Sigma Notation to write the sum $\frac{4}{5-1} + \frac{5}{5-8} + \frac{6}{5-27} + \dots + \frac{9}{5-216}$

3) Use properties of summation to evaluate the sum:

$$\sum_{i=1}^{31} 1 - 3(i-1)^2$$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis: $f(x) = 2 + 5x^2$ in interval $[1, 8]$ using 3 rectangles

a) Left endpoint (LRAM):

b) Right Endpoint(RRAM):

c) Middle rectangle (MRAM):

5) Selected values of a function, f , are given in the table below:

x	2	4	7	9	16	17	21	23	25
$f(x)$	6	1	2	4	6	7	12	7	10

- a) Use middle rectangles to approximate the area with 3 subintervals for f on the interval interval $[4, 23]$

x	2	4	7	9	16	17	21	23	25
$f(x)$	6	1	2	4	6	7	12	7	10

- b) Use right-handed rectangles to approximate the area with 4 subintervals for f on the interval $[2, 25]$

x	2	4	7	9	16	17	21	23	25
$f(x)$	6	1	2	4	6	7	12	7	10

- c) Use left-handed rectangles to approximate the area with 2 subintervals for f on the interval $[7, 25]$

x	2	4	7	9	16	17	21	23	25
$f(x)$	6	1	2	4	5	7	12	7	10

- d) Use trapezoids to approximate the area with 2 subintervals for f on the interval $[2, 25]$

Key

1) Find the sum $\sum_{k=4}^7 \frac{k}{3^k - k^2}$

$$\frac{4}{3^4 - 4^2} + \frac{5}{3^5 - 5^2} + \frac{6}{3^6 - 6^2} + \frac{7}{3^7 - 7^2} = \frac{4}{65} + \frac{5}{218} + \frac{6}{231} + \frac{7}{2138} \approx 0.096$$

2) Use Sigma Notation to write the sum $\sum_{i=1}^4 \frac{i+3}{5-i^3}$

$$\sum_{i=1}^6 \frac{i+3}{5-i^3}$$

3) Use properties of summation to evaluate the sum:

$$\sum_{i=1}^{31} 1 - 3(i-1)^2 \rightarrow 1 - 3(i-1)(i-1) \rightarrow 1 - 3(i^2 - 2i + 1) \rightarrow 1 - 3i^2 + 6i - 3 \rightarrow \sum -3i^2 + 6i - 2$$

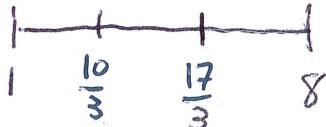
$$\sum -3i^2 + \sum 6i - \sum 2$$

$$-3 \left[\sum i^2 \right] + 6 \left[\sum i \right] - 2 \left[\sum 1 \right]$$

$-3 \left[\frac{n(n+1)(2n+1)}{6} \right] + 6 \left[\frac{n(n+1)}{2} \right] - 2[n]$	-31248
$-3 \left[\frac{31(32)(63)}{6} \right] + 6 \left[\frac{31(32)}{2} \right] - 2[31]$	$+2976$
	-62
	$-28,334$

4) Use Left, Middle, and Right endpoints to find approximations of area of region between the graph and the x-axis: $f(x) = 2 + 5x^2$ in interval $[1, 8]$ using 3 rectangles

$$W = \frac{b-a}{n} = \frac{8-1}{3} = \frac{7}{3}$$



a) Left endpoint (LRAM):

$$A = \frac{7}{3} \left[f(1) + f\left(\frac{10}{3}\right) + f\left(\frac{17}{3}\right) \right]$$

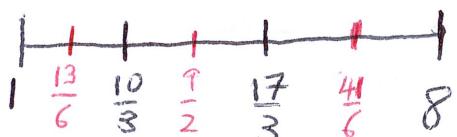
$$A = \frac{7}{3} \left[7 + \frac{518}{9} + \frac{1463}{9} \right] = \frac{14308}{27} \approx 529.926$$

b) Right Endpoint(RRAM):

$$A = \frac{7}{3} \left[f\left(\frac{10}{3}\right) + f\left(\frac{17}{3}\right) + f(8) \right]$$

$$\frac{7}{3} \left[\frac{518}{9} + \frac{1463}{9} + 322 \right] \approx 1264.926$$

c) Middle rectangle (MRAM):



$$A \approx \frac{7}{3} \left[f\left(\frac{13}{6}\right) + f\left(\frac{9}{2}\right) + f\left(\frac{41}{6}\right) \right]$$

$$\approx \frac{849.787}{108} = \frac{91777}{108}$$

5) Selected values of a function, f , are given in the table below:

	5			8			6		
x	2	4	7	9	16	17	21	23	25
f(x)	6	1	(2)	4	(6)	7	(12)	7	10

$$\text{Area} = \text{width} \times \text{height}$$

- a) Use middle rectangles to approximate the area with 3 subintervals for f on the interval interval $[4, 23]$

$$\begin{aligned}\text{Area} &\approx 5 \cdot f(7) + 8 \cdot f(16) + 6 \cdot f(21) \\ &= 5(2) + 8(6) + 6(12) = 130\end{aligned}$$

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	(2)	4	(6)	7	(12)	7	(10)

- b) Use right-handed rectangles to approximate the area with 4 subintervals for f on the interval $[2, 25]$

$$\begin{aligned}\text{Area} &\approx 5 \cdot f(7) + 9 \cdot f(16) + 5 \cdot f(21) + 4 \cdot f(25) \\ &= 5(2) + 9(6) + 5(12) + 4(10) = 164\end{aligned}$$

x	2	4	7	9	16	17	21	23	25
f(x)	6	1	(2)	4	6	(7)	12	7	10

- c) Use left-handed rectangles to approximate the area with 2 subintervals for f on the interval $[7, 25]$

$$\begin{aligned}\text{Area} &\approx 10 \cdot f(7) + 8 \cdot f(17) \\ &= 10(2) + 8(7) = 76\end{aligned}$$

x	2	4	7	9	16	17	21	23	25
f(x)	(6)	1	2	4	(5)	7	12	7	(10)

$$\text{Area} = \frac{w}{2}[h_1 + h_2]$$

- d) Use trapezoids to approximate the area with 2 subintervals for f on the interval $[2, 25]$

$$\begin{aligned}\text{Area} &\approx \frac{14}{2}[f(2) + f(16)] + \frac{9}{2}[f(16) + f(25)] \\ &= 7[6 + 5] + \frac{9}{2}[5 + 10] = 144.5\end{aligned}$$