

5. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on $[0, 7]$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on $[0, 10]$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on $[4, 10]$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on $[0, 7]$

6. Given the region bounded by $g(x) = 6 - x^2$, the x-axis, $x = -1$, and $x = 2$. Use the limit definition to find the exact area of the region.

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{b-a}{n} \right) \cdot f \left[a + \left(\frac{b-a}{n} \right) i \right]$$

$$A = \lim_{n \rightarrow \infty} \sum \left(\frac{3}{n} \right) \cdot f \left[-1 + \frac{3}{n} i \right]$$

$$\frac{3}{n} \cdot \left[6 - \left(-1 + \frac{3}{n} i \right)^2 \right]$$

$$\frac{3}{n} \left[6 - \left(\frac{9}{n^2} i^2 - \frac{6}{n} i + 1 \right) \right]$$

$$\frac{3}{n} \left[6 - \frac{9}{n^2} i^2 + \frac{6}{n} i - 1 \right]$$

$$\frac{3}{n} \left[5 - \frac{9}{n^2} i^2 + \frac{6}{n} i \right]$$

$$\sum \frac{15}{n} - \sum \frac{27}{n^3} i^2 + \sum \frac{18}{n^2} i$$

$$\frac{15}{n} \left[\sum 1 \right] - \frac{27}{n^3} \left[\sum i^2 \right] + \frac{18}{n^2} \left[\sum i \right]$$

$$\lim_{n \rightarrow \infty} \frac{15}{n} (n) - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{18}{n^2} \cdot \frac{n(n+1)}{2}$$

$$\lim_{n \rightarrow \infty} \frac{15n}{n} - \frac{54n^3 \dots}{6n^3} + \frac{18n^2 \dots}{2n^2}$$

$$15 - \frac{54}{6} + \frac{18}{2} = \boxed{15}$$