Ch. 4.2 Sigma Notation p. 263-264 #1-19 odd, 31, 33, 35

a) $\sum_{i=1}^{n} 1 = n$

b) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

c) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

d) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Evaluate a Sum

15) $\sum_{i=1}^{24} 4i = 4 \cdot \sum_{i=1}^{24} i = 4 \cdot \frac{24(25)}{2} = \boxed{1,200}$

17) $\sum_{i=1}^{20} (i-1)^2 = \sum i^2 - 2i + 1 = \sum_{i=1}^{20} i^2 - 2\sum_{i=1}^{20} i + \sum_{i=1}^{20} 1$

$$= \frac{20(21)(41)}{6} - 2\left(\frac{20(21)}{2}\right) + 20 = \boxed{2,470}$$

19) $\sum_{i=1}^{15} i(i-1)^2 = \sum i(i^2 - 2i + 1) = \sum i^3 - 2i^2 + i$

$$= \sum_{i=1}^{15} i^3 - 2\sum_{i=1}^{15} i^2 + \sum_{i=1}^{15} i = \frac{15^2(16)^2}{4} - 2 \cdot \left(\frac{15(16)(31)}{6}\right) + \frac{15(16)}{2}$$

$$= \boxed{12,040}$$

Ch. 4.2c Riemann Sums, Exact Area under Curve Limit Definition

p. 263-264 #37, 39, 41, 45, 46, 49

37) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{24i}{n^2} = \frac{24}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty} \frac{24}{n^2}\left[\frac{n(n+1)}{2}\right] = \lim_{n\to\infty} \frac{24n^2+24n}{2n^2} = \boxed{12}$

39) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2 = \frac{1}{n^3}\sum i^2-2i+1 = \frac{1}{n^3}\sum i^2 - \frac{2}{n^3}\sum i + \frac{1}{n^3}\sum 1$

$= \lim_{n\to\infty} \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{2}{n^3}\cdot\frac{n(n+1)}{2} + \frac{1}{n^3}(n)$

$= \lim_{n\to\infty} \frac{2n^3+2n^2}{6n^3} - \frac{2n^2+2n}{2n^3} + \frac{n}{n^3} = \frac{2}{6} - 0 + 0 = \boxed{\frac{1}{3}}$

41) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \sum \frac{2}{n}+\frac{2i}{n^2} = \lim_{n\to\infty} \frac{2}{n}\sum 1 + \frac{2}{n^2}\sum i$

$= \lim_{n\to\infty} \frac{2}{n}(n) + \frac{2}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty} \frac{2n}{n} + \frac{2n^2+2n}{2n^2}$

$= 2 + 1 = \boxed{3}$

45) Find Area by limit definition: $\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width}\cdot f[a+\text{width}\cdot i]$ $\quad \text{width} = \frac{b-a}{n}$

$y = -4x+5 \quad [\overset{a}{0}, \overset{b}{1}] \leftarrow \text{width} = \frac{1-0}{n} = \frac{1}{n}$

$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\cdot f\left[0+\frac{1}{n}i\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[-4\left(\frac{1}{n}i\right)+5\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{-4}{n^2}i + \frac{5}{n}$

$= \lim_{n\to\infty} \frac{-4}{n^2}\sum i + \frac{5}{n}\sum 1$

$= \lim_{n\to\infty} \frac{-4}{n^2}\cdot\frac{n(n+1)}{2} + \frac{5}{n}\cdot n$

$= \lim_{n\to\infty} \frac{-4n^2-4}{2n^2} + \frac{5n}{n} = -2+5 = \boxed{3}$ Area

$\boxed{4.2c}$ $\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width} \cdot f[a + \text{width} \cdot i]$

46) $y = 3x - 2 \quad [2,5]$ $\quad \text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \cdot f\left[2 + \frac{3}{n} i\right]$

$\text{width} = \frac{5-2}{n} = \frac{3}{n}$

$A = \lim_{n\to\infty} \frac{3}{n} \cdot \left[3\left[2 + \frac{3}{n}i\right] - 2\right]$

$= \lim_{n\to\infty} \sum \frac{3}{n}\left(6 + \frac{9}{n}i - 2\right)$

$= \lim_{n\to\infty} \sum \frac{3}{n}\left(4 + \frac{9}{n}i\right)$

$\lim_{n\to\infty} \sum \frac{12}{n} + \frac{27}{n^2} i$

$\lim_{n\to\infty} \frac{12}{n} \sum 1 + \frac{27}{n^2} \sum i$

$\lim_{n\to\infty} \frac{12}{n}(n) + \frac{27}{n^2} \cdot \frac{n(n+1)}{2}$

$\lim_{n\to\infty} \frac{12n}{n} + \frac{27n^2 + 27n}{2n^2}$

$= 12 + \frac{27}{2} = 12 + 13.5$

$= \boxed{25.5 = \frac{51}{2}}$

49) $y = 25 - x^2 \quad [1,4]$ $\quad \text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \cdot f\left[1 + \frac{3}{n} i\right]$

$\text{width} = \frac{4-1}{n} = \frac{3}{n}$

$A = \lim_{n\to\infty} \frac{3}{n} \cdot \left[25 - \left(1 + \frac{3}{n}i\right)^2\right]$

$= \lim_{n\to\infty} \sum \frac{3}{n}\left[25 - \left(1 + \frac{6i}{n} + \frac{9}{n^2}i^2\right)\right]$

$= \lim_{n\to\infty} \sum \frac{3}{n}\left[25 - 1 - \frac{6i}{n} - \frac{9}{n^2}i^2\right]$

$= \lim_{n\to\infty} \sum \frac{3}{n}\left[24 - \frac{6i}{n} - \frac{9}{n^2}i^2\right]$

$= \lim_{n\to\infty} \sum \frac{72}{n} - \frac{18i}{n^2} - \frac{27}{n^3} i^2$

$= \lim_{n\to\infty} \frac{72}{n} \sum 1 - \frac{18}{n^2} \sum i - \frac{27}{n^3} \sum i^2$

$= \lim_{n\to\infty} \frac{72}{n}(n) - \frac{18}{n^2} \cdot \frac{n(n+1)}{2} - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$ $\quad n(2n^2+3n+1)$

$= \lim_{n\to\infty} \frac{72n}{n} - \frac{18n^2 - 18n}{2n^2} - \frac{54n^3 - 81n^2 - 27n}{6n^3}$

$= 72 - \frac{18}{2} - \frac{54}{6} = 72 - 9 - 9 = \boxed{54}$