

②

Key

Linear Approximation Practice Problems

Use the given information and a linear approximation to approximate the value of the function at c

Linear approximation steps:

i) Find the tangent line equation ii) plug in decimal into linear equation.

1) $f(2) = 8; f'(2) = -3; c = 2.06$ ↖ decimal value

point: $(2, 8)$
 slope: $m = -3$

$$y - 8 = -3(x - 2) \quad \left| \begin{array}{l} y = -3(x - 2) + 8 \\ y(2.06) = -3(2.06 - 2) + 8 \\ = \boxed{7.82} \end{array} \right.$$

2) $f(-4) = 3; f'(-4) = 2; c = -3.6$

point: $(-4, 3)$
 slope: $m = 2$

$$y - 3 = 2(x + 4) \quad \left| \begin{array}{l} y = 2(x + 4) + 3 \\ y(-3.6) = 2(-3.6 + 4) + 3 \\ \boxed{y(-3.6) = 3.8} \end{array} \right.$$

3) $f(-1) = 0; f'(-1) = \frac{3}{2}; c = -1.1$

point: $(-1, 0)$
 slope: $m = \frac{3}{2}$

$$y - 0 = \frac{3}{2}(x + 1) \quad \left| \begin{array}{l} y(-1.1) = \frac{3}{2}(-1.1 + 1) = \boxed{-0.15} \end{array} \right.$$

4) $f(5) = \frac{1}{2}; f'(5) = -3; c = 5.2$

point: $(5, \frac{1}{2})$
 slope: $m = -3$

$$y - \frac{1}{2} = -3(x - 5) \quad \left| \begin{array}{l} y = -3(x - 5) + \frac{1}{2} \\ y(5.2) = -3(5.2 - 5) + \frac{1}{2} \\ \boxed{y(5.2) = -0.1} \end{array} \right.$$

Linear approximation steps:

- i) Find the tangent line equation using nearest integer
- ii) plug in decimal into linear equation.

5) Given $f(x) = 2x^3 - 4x + 1$ Use Linear approximation to approximate $f(1.02)$

at $x=1$

$$f(1) = 2 - 4 + 1 = -1$$

$$f'(x) = 6x^2 - 4$$

$$f'(1) = 6 - 4 = 2$$

point: $(1, -1)$
slope: $m = 2$
 $y + 1 = 2(x - 1)$

$$y = 2(x - 1) - 1$$

$$y(1.02) = 2(1.02 - 1) - 1$$

$$y(1.02) = -0.96$$

← quotient

6) Given $f(x) = \frac{2x^3 - 1}{3 - x}$ Use Linear approximation to approximate $f(1.01)$

at $x=1$

$$f(1) = \frac{2 - 1}{3 - 1} = \frac{1}{2}$$

$$f'(x) = \frac{(6x^2)(3 - x) - (2x^3 - 1)(-1)}{(3 - x)^2}$$

$$f'(1) = \frac{6(2) - (1)(-1)}{(3 - 1)^2} = \frac{13}{4}$$

point: $(1, 1/2)$
slope: $m = 13/4$
 $y - 1/2 = \frac{13}{4}(x - 1)$
 $y = \frac{13}{4}(x - 1) + 1/2$
 $y = \frac{13}{4}(1.01 - 1) + 1/2 = 0.5325$

7) Given $f(x) = \sqrt[3]{3x + 2}$ Use Linear approximation to approximate $f(2.1)$

at $x=2$

$$f(x) = (3x + 2)^{1/3}$$

$$f(2) = (3(2) + 2)^{1/3} = \sqrt[3]{8} = 2$$

$$f'(x) = \frac{1}{3}(3x + 2)^{-2/3} \cdot (3)$$

$$f'(2) = \frac{1}{(3(2) + 2)^{2/3}} = \frac{1}{8^{2/3}} = \frac{1}{4}$$

point: $(2, 2)$ slope: $m = 1/4$
 $y - 2 = \frac{1}{4}(x - 2)$
 $y = \frac{1}{4}(x - 2) + 2$
 $y(2.1) = \frac{1}{4}(2.1 - 2) + 2 = 2.025$

out: $()^{1/3}$
in: $3x + 2$

$$f'(x) = \frac{1}{(3x + 2)^{2/3}}$$