Riemann Sum Worksheet **Name** ____________________

For each problem sketch the graph showing the appropriate region, then approximate the area bound by the curve and the x-axis on the given interval using 6 different Riemann sums Left, Right, Upper, Lower, midpoint, Trapezoidal, using the specified number of subintervals.

1. Function : $f(x) = -(x-3)^2+20$ on Interval [0, 5] using 5 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

2. Function : $f(x)$ =2sinx+3 on Interval $[0, 2\pi]$using 6 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

3. Function : $f(x) = \sqrt[3]{2x-1} + 5$ on Interval [-2, 2] using 4 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

4.

x	-4	-2	0	3	6	13	20
f(x)	8	12	18	4	9	31	12

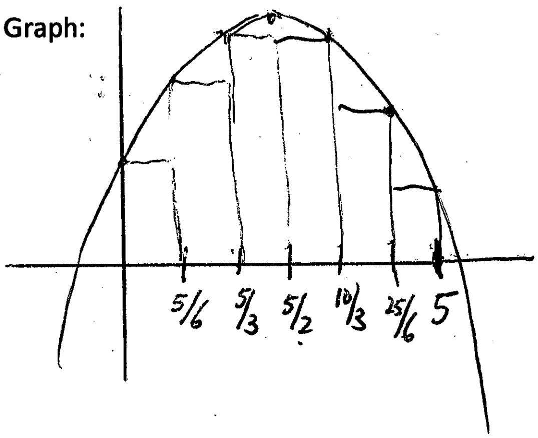
Left Sum- 6 Subintervals	Right Sum- 6 Subintervals
Lower Sum- 6 Subintervals	Upper Sum- 6 Subintervals
Midpoint Sum- 3 Subintervals	Trapezoidal Sum- 6 Subintervals

Riemann Sum Worksheet **Name** Key

For each problem sketch the graph showing the appropriate region, then approximate the area bound by the curve and the x-axis on the given interval using 6 different Riemann sums Left, Right, Upper, Lower, midpoint, Trapezoidal, using the specified number of subintervals.

1. Function : $f(x) = -(x-3)^2+20$ on Interval [0, 5] using 5 subintervals

Graph:



width $= \frac{b-a}{n} = \frac{5-0}{6} = 5/6$

Left Sum

$\frac{5}{6}\left[f(0)+f(5/6)+f(5/3)+f(5/2)+f(10/3)+f(25/6)\right]$

Right Sum

$\frac{5}{6}\left[f(5/6)+f(5/3)+f(5/2)+f(10/3)+f(25/6)+f(5)\right]$

Lower Sum

$\frac{5}{6}\left[f(0)+f(5/6)+f(5/3)+f(10/3)+f(25/6)+f(5)\right]$

Upper Sum

$\frac{5}{6}\left[f(5/6)+f(5/3)+f(5/2)+f(5/2)+f(10/3)+f(25/6)\right]$

Midpoint Sum

$\frac{5}{6}\left[f(5/12)+f(5/4)+f(25/12)+f(35/12)+f(15/4)+f(55/12)\right]$

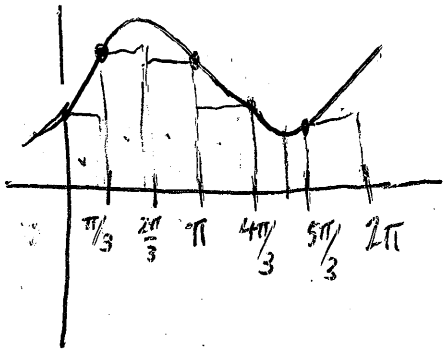
Trapezoidal Sum

$\frac{w}{2}\left[h_1+2h_2+2h_3+\dots h_n\right]$

$\frac{5}{6}\cdot\frac{1}{2}\left[f(0)+2f(5/6)+2f(5/3)+2f(5/2)+2f(10/3)+2f(25/6)+f(5)\right]$

2. Function : $f(x)=2\sin x+3$ on Interval $[0, 2\pi]$ using 6 subintervals $\frac{b-a}{n}=\frac{2\pi-0}{6}=\frac{\pi}{3}$

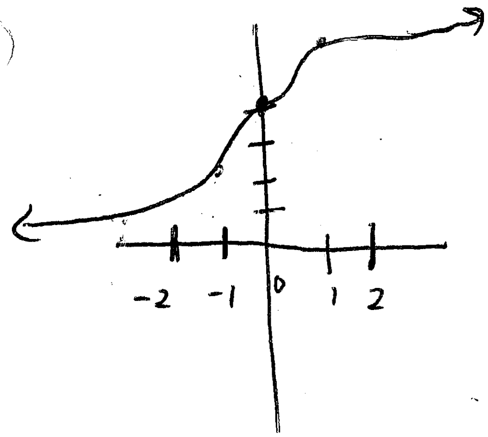
Graph:



Left Sum $\frac{\pi}{3}\left[f(0)+f(\pi/3)+f(2\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(5\pi/3)\right]$	**Right Sum** $\frac{\pi}{3}\left[f(\pi/3)+f(\frac{2\pi}{3})+f(\pi)+f(4\pi/3)+f(5\pi/3)+f(2\pi)\right]$
Lower Sum $\frac{\pi}{3}\left[f(0)+f(\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(5\pi/3)+f(2\pi)\right]$	**Upper Sum** $\frac{\pi}{3}\left[f(\pi/3)+f(\frac{2\pi}{3})+f(2\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(2\pi)\right]$
Midpoint Sum $\pi/3\left[f(\pi/6)+f(\pi/2)+f(7\pi/6)+f(3\pi/2)+f(11\pi/6)\right]$	**Trapezoidal Sum** $\pi/3\left[f(0)+2f(\pi/3)+2f(2\pi/3)+2f(\pi)+2f(\frac{4\pi}{3})+2f(5\pi/3)+f(2\pi)\right]$

3. Function : $f(x) = \sqrt[3]{2x-1} + 5$ on Interval [-2, 2] using 4 subintervals $W = \frac{b-a}{n} = \frac{2--2}{4} = 1$

Graph:

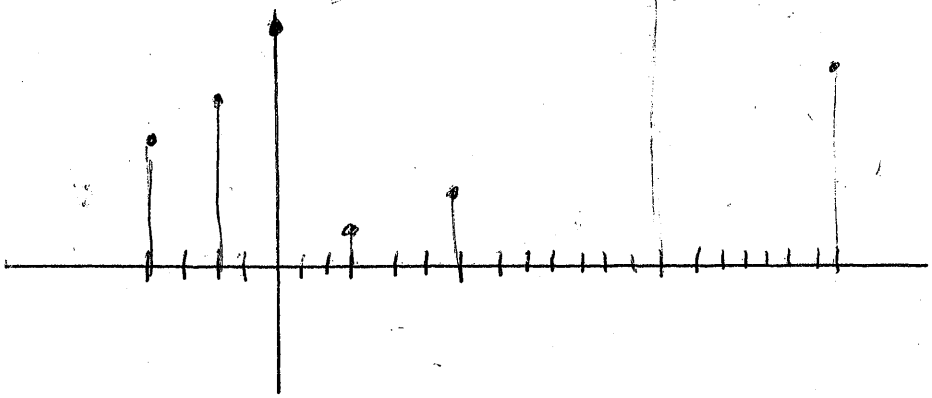


Left Sum	Right Sum
$1\left[f(-2) + f(-1) + f(0) + f(1)\right]$	$1\left[f(-1) + f(0) + f(1) + f(2)\right]$
Lower Sum	**Upper Sum**
$1\left[f(-2) + f(-1) + f(0) + f(1)\right]$	$1\left[f(-1) + f(0) + f(1) + f(2)\right]$
Midpoint Sum	**Trapezoidal Sum**
$1\left[f(-1.5) + f(-\tfrac{1}{2}) + f(\tfrac{1}{2}) + f(1.5)\right]$	$\frac{1}{2} \cdot 1\left[f(-2) + 2f(-1) + 2f(0) + 2f(1) + f(2)\right]$

4.

1 2 3 4 5 6

x	−4	−2	0	3	6	13	20
f(x)	8	12	18	4	9	31	12



Left Sum- 6 Subintervals

$2f(-4)+2(f(-2))+3f(0)+3f(3)+7f(6)+7f(13)$

$2(8)+2(12)+3(18)+3(4)+7(9)+7(31)$

Right Sum- 6 Subintervals

$2f(-2)+2f(0)+3f(3)+3f(6)+7f(13)+7f(20)$

Lower Sum- 6 Subintervals

$2f(-4)+2f(-2)+3f(3)+3f(3)+7f(6)+7f(20)$

Upper Sum- 6 Subintervals

$2f(-2)+2f(0)+3f(0)+3f(6)+7f(13)+7f(13)$

Midpoint Sum- 3 Subintervals

$4f(-2)+6f(3)+14f(13)$

$4(12)+6(4)+14(13)$

Trapezoidal Sum- 6 Subintervals

$\frac{2}{2}[f(-4)+f(-2)]+\frac{2}{2}[f(-2)+f(0)]$

$+\frac{3}{2}[f(0)+f(3)]+\frac{3}{2}[f(3)+f(6)]$

$+\frac{7}{2}[f(6)+f(13)]+\frac{7}{2}[f(13)+f(20)]$