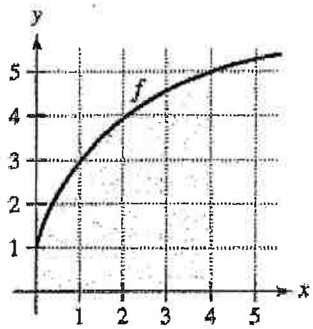
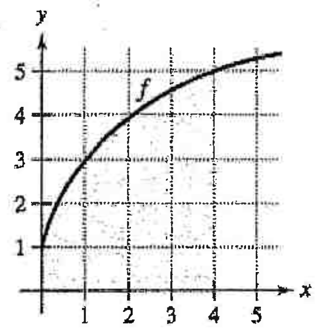
Ch. 4.2 Homework Problems

Using Upper and Lower Sums In Exercises 31 and 32, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

31.

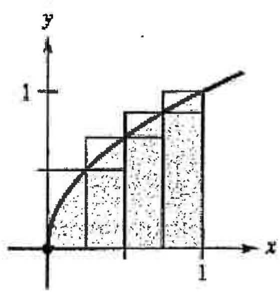


31.

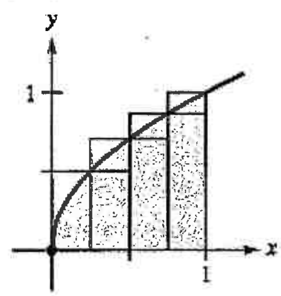


Finding Upper and Lower Sums for a Region In Exercises 33–36, use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width).

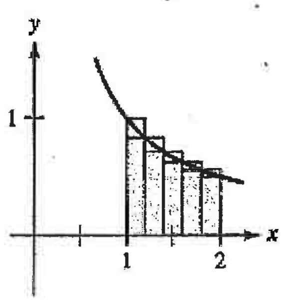
33. $y = \sqrt{x}$



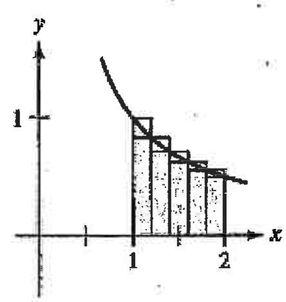
33. $y = \sqrt{x}$



35. $y = \dfrac{1}{x}$



35. $y = \dfrac{1}{x}$



Approximating the Area of a Plane Region In Exercises 25–30, use left and right endpoints and the given number of rectangles to find two approximations of the area of the region between the graph of the function and the x-axis over the given interval.

25. $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles

27. $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles

29. $f(x) = \cos x$, $\left[0, \frac{\pi}{2}\right]$, 4 rectangles

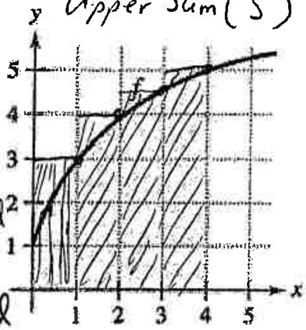
2017

Key

Ch. 4.2 Homework Problems

Using Upper and Lower Sums In Exercises 31 and 32, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

31. Upper Sum (S)



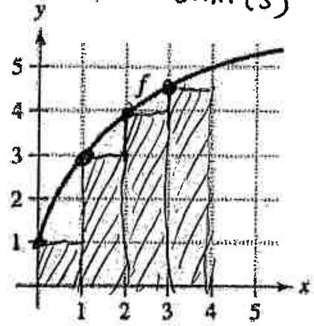
Since $f(x)$ is always increasing upper sum is right-handed rectangles (circumscribed rectangles)

$S = 1f(1) + 1f(2) + 1f(3) + 1f(4)$

$= 1(3) + 1(4) + 1(4.5) + 1(5)$

$S = \boxed{\frac{33}{2} \text{ or } 16.5}$

31. lower sum (s)



since $f(x)$ is increasing, lower sum is left-handed rectangles (inscribed rectangles)

$s = 1f(0) + 1f(1) + 1f(2) + 1f(3)$

$s = 1(1) + 1(3) + 1(4) + 1(4.5)$

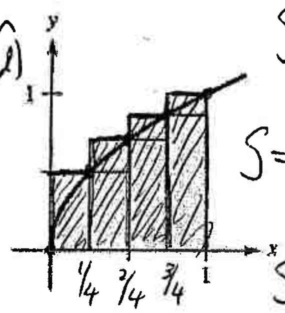
$s = \boxed{\frac{25}{2} = 12.5}$

Finding Upper and Lower Sums for a Region In Exercises 33–36, use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width).

width $= \frac{b-a}{n} = \frac{1-0}{4} = 1/4$

33. $y = \sqrt{x}$

upper sum (right-handed)



$S = \frac{1}{4} \cdot y(1/4) + \frac{1}{4} \cdot y(2/4) + \frac{1}{4} y(3/4) + 1/4\, y(1)$

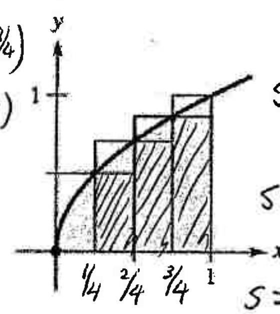
$S = \frac{1}{4}\left[\sqrt{1/4} + \sqrt{2/4} + \sqrt{3/4} + \sqrt{1}\right]$

$S = \frac{1}{4}\left[\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 1\right]$

$\boxed{S \approx 0.768}$

33. $y = \sqrt{x}$

lower sum (left-handed)



$s = \frac{1}{4}\left[y(0) + y(1/4) + y(2/4) + y(3/4)\right]$

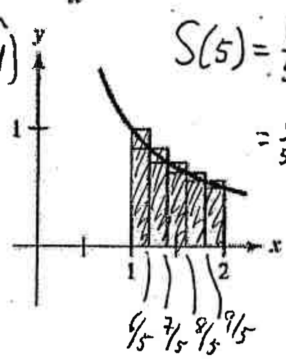
$s = \frac{1}{4}\left[0 + \sqrt{1/4} + \sqrt{2/4} + \sqrt{3/4}\right]$

$s = \frac{1}{4}\left[\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right]$

$\boxed{s \approx 0.518}$

35. $y = \frac{1}{x}$ width $= \frac{b-a}{n} = \frac{2-1}{5} = \frac{1}{5}$

upper sum (left-handed)



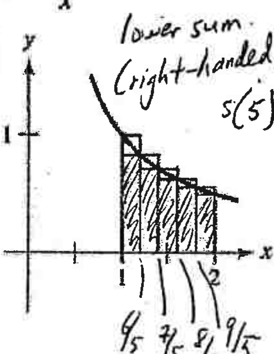
$S(5) = \frac{1}{5}\left[y(1) + y(6/5) + y(7/5) + y(8/5) + y(9/5)\right]$

$= \frac{1}{5}\left[\frac{1}{1} + \frac{1}{6/5} + 1/\tfrac{7}{5} + 1/\tfrac{8}{5} + 1/\tfrac{9}{5}\right]$

$\boxed{S(5) \approx 0.746}$

35. $y = \frac{1}{x}$

lower sum (right-handed)



$s(5) = \frac{1}{5}\left[y(6/5) + y(7/5) + y(8/5) + y(9/5) + y(1)\right]$

≈ 0.646

$\boxed{s(5) \approx 0.646}$

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Ch. 4.2 Continued

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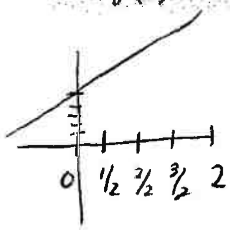
Approximating the Area of a Plane Region In Exercises 25–30, use left and right endpoints and the given number of rectangles to find two approximations of the area of the region between the graph of the function and the x-axis over the given interval.

$\text{width} = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

25. $f(x) = 2x + 5$, $[0, 2]$ (a, b), 4 rectangles ($n = 4$)



Left endpt: $A = \frac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right] = \frac{1}{2}\left[5+6+7+8\right] = \frac{26}{2} = \boxed{13}$

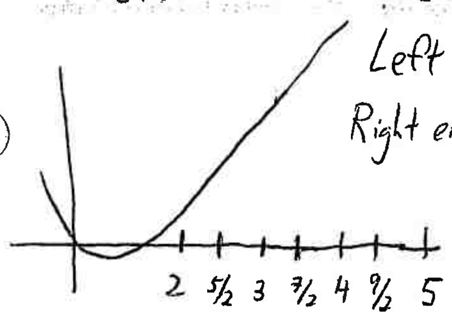
Right endpt: $A = \frac{1}{2}\left[f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right] = \frac{1}{2}(6+7+8+9) = \frac{30}{2} = \boxed{15}$

$\text{width} = \frac{b-a}{n} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$

27. $g(x) = 2x^2 - x - 1$, $[2, 5]$ (a, b), 6 rectangles ($n = 6$)

Left endpt: $A = \frac{1}{2}\left[f(2) + f(\tfrac{5}{2}) + f(3) + f(\tfrac{7}{2}) + f(4) + f(\tfrac{9}{2})\right] = \boxed{55}$

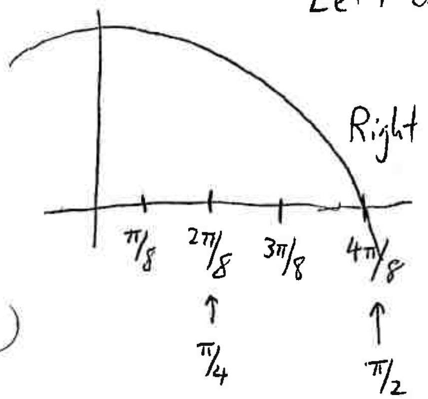
Right endpt: $A = \frac{1}{2}\left[f(\tfrac{5}{2}) + f(3) + f(\tfrac{7}{2}) + f(4) + f(\tfrac{9}{2}) + f(5)\right] = \boxed{74.5}$



$\text{width} = \frac{b-a}{n} = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$

29. $f(x) = \cos x$, $\left[0, \frac{\pi}{2}\right]$ (a, b), 4 rectangles ($n = 4$)

Left endpt: $\frac{\pi}{8}\left[f(0) + f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8})\right] \approx \boxed{1.1835}$



Right endpt: $\frac{\pi}{8}\left[f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8}) + f(\tfrac{\pi}{2})\right] \approx \boxed{0.7908}$