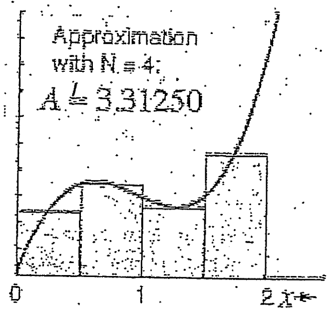
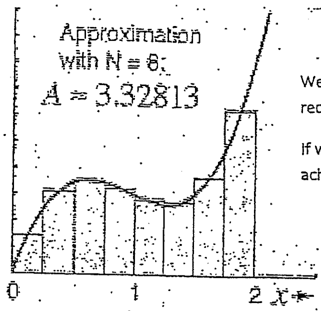
4.2c Finding Exact Area using limits





We can continually improve the Area Approximation under the curve by increasing the number of rectangles : above (n = 4) and n = 8, ...n = 16...

If we let n go out to infinity, (using limits) we will have something better than an approximation, we will achieve the actual area under the curve.

$$\text{Exact Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{b-a}{n}\right) \cdot f\left(a + \left(\frac{b-a}{n}\right) i\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (\text{width}) \cdot f\left(\underline{\text{left endpoint}} + \underline{\text{width} \cdot i}\right)$$

Memorize → "width f left plus width times i"

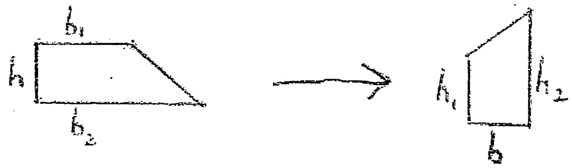
$\boxed{\text{Ex. 1}}$ Find exact area between $f(x) = 4 - x^2$ and x-axis from $[-2, 2]$

4.6 Trapezoids

* Better approximation than inscribed, circumscribed, or midpoint rectangles.

Trapezoidal Rule: Approximate Area of region using areas of trapezoids.

Review: Area of Trapezoid $= \frac{1}{2}h(b_1+b_2)$ or $\frac{h}{2}(b_1+b_2)$



$\boxed{\text{Area} = \frac{b}{2}(h_1+h_2)}$ or $\frac{1}{2}w(h_1+h_2)$

Ex. 1 Estimate area bounded by $f(x) = 6x - x^2$ and the x-axis using 6 trapezoids.