Limit Definition of Area Practice Problems WS

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left[\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]\right]$$

1) $y = 2x^2 - 3x + 2$ $\quad [1, 3]$

2) $y = 1 - 2x - x^2$ $[-1, 4]$

Use Limit Definition of Area: $\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]\right]$

1) $y = 2x^2 - 3x + 2 \qquad [1, 3]$

$\text{width} = \frac{3-1}{n} = \frac{2}{n}$

$\lim_{n\to\infty} \sum \frac{2}{n} \cdot f\left[1 + \frac{2}{n} i\right]$

$\lim_{n\to\infty} \sum \frac{2}{n} \left[2\left(1+\frac{2}{n}i\right)^2 - 3\left(1+\frac{2}{n}i\right) + 2\right]$

$\frac{2}{n}\left[2\left(1+\frac{2}{n}i\right)\left(1+\frac{2}{n}i\right) - 3\left(1+\frac{2}{n}i\right) + 2\right]$

$\frac{2}{n}\left[2\left(1+\frac{2}{n}i + \frac{2}{n}i + \frac{4}{n^2}i^2\right) - 3\left(1+\frac{2}{n}i\right) + 2\right]$

$\frac{2}{n}\left[2\left(1+\frac{4}{n}i + \frac{4}{n^2}i^2\right) - 3\left(1+\frac{2}{n}i\right) + 2\right]$

$\frac{2}{n}\left[2 + \frac{8}{n}i + \frac{8}{n^2}i^2 - 3 - \frac{6}{n}i + 2\right]$

$\sum \frac{2}{n}\left[\frac{8}{n^2}i^2 + \frac{2}{n}i + 1\right]$

$\sum \frac{16}{n^3}i^2 + \frac{4}{n^2}i + \frac{2}{n}$

$\sum \frac{16}{n^3}i^2 + \sum \frac{4}{n^2}i + \sum \frac{2}{n}$

$\frac{16}{n^3}\boxed{\sum i^2} + \frac{4}{n^2}\boxed{\sum i} + \frac{2}{n}\boxed{\sum 1}$

$\lim_{n\to\infty} \frac{16}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{4}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{2}{n}[n]$

$\lim_{n\to\infty} \frac{32n^3 + \ldots}{6n^3} + \frac{4n^2 + \ldots}{2n^2} + \frac{2n}{n}$

$\frac{32}{6} + 2 + 2 = \boxed{\frac{28}{3}}$

2) $y = 1 - 2x - x^2 \qquad [-1, 4]$

$$\text{width} = \frac{4--1}{n} = \frac{5}{n}$$

$$\lim_{n\to\infty} \sum \frac{5}{n} \cdot f\left[-1 + \frac{5}{n}i\right]$$

$$\sum \frac{5}{n} \cdot \left[1 - 2\left(-1 + \frac{5}{n}i\right) - \left(-1 + \frac{5}{n}i\right)^2\right]$$

$$\sum \frac{5}{n} \left[1 + 2 - \frac{10}{n}i - \left(-1 + \frac{5}{n}i\right)\left(-1 + \frac{5}{n}i\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - \left(1 - \frac{5}{n}i - \frac{5}{n}i + \frac{25}{n^2}i^2\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - \left(1 - \frac{10}{n}i + \frac{25}{n^2}i^2\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - 1 + \frac{10}{n}i - \frac{25}{n^2}i^2\right]$$

$$\sum \frac{5}{n} \left[2 - \frac{25}{n^2}i^2\right]$$

$$\sum \frac{10}{n} - \frac{125}{n^3}i^2 = \sum \frac{10}{n} - \sum \frac{125}{n^3}i^2$$

$$\frac{10}{n} \boxed{\sum 1} - \frac{125}{n^3} \boxed{\sum i^2}$$

$$\lim_{n\to\infty} \frac{10}{n}[n] - \frac{125}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{10n}{n} - \frac{250n^3 + \dots}{6n^3}$$

$$10 - \frac{250}{6} = \boxed{-\frac{95}{3}}$$