35pts

A.P. Calculus AB **Quiz 4-3, 4-4, 4-5**

NO CALCULATORS!

Justify your answers and show your work.

Name: Key A Date: 1/27/2017 (Fri) Per.

5 pts each

1. Money is withdrawn from an account in such a manner that the amount of money A (in thousands of dollars) in the account at time t years is given by the equation:

$$\frac{1}{4-0}\int_0^4 16-4t^{1/2}\,dt$$

$$A(t) = 16 - 4\sqrt{t} \text{ for } t \geq 0.$$

At what time does the account have the average amount of money in it during the first 4 years.

$$\frac{1}{4}\left(16t-\frac{4t^{3/2}}{3/2}\right) \qquad 4t-\frac{2}{3}t^{3/2}\Big]_0^4 \to 16-\frac{2}{3}(4)^{3/2}-(0-0)$$

$$16-\frac{2}{3}(8) = \frac{16}{1}-\frac{16}{3} = \frac{48}{3}-\frac{16}{3}$$

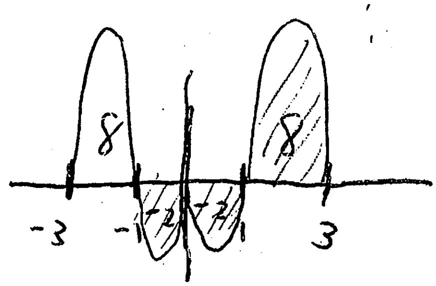
Answer: $A(t)=\frac{32}{3}$, $\boxed{t=\frac{16}{9}\text{ yrs}}$

$$\frac{32}{3}=16-4\sqrt{t} \qquad \frac{32}{3}-16=-4\sqrt{t} \qquad -\frac{16}{3}=-4\sqrt{t} \qquad \boxed{\frac{4}{3}=\sqrt{t}}$$

2. If $f(x)=\int_{5x^3}^{-3}\frac{7}{\sqrt{1-t^2}}\,dt$, find $\frac{d}{dx}f(x)$.

Answer: $\boxed{\dfrac{-105x^2}{\sqrt{1-25x^6}}}$

$$\frac{d}{dx}\left[-\int_{-3}^{5x^3}\frac{7}{\sqrt{1-t^2}}\right] = -\frac{7}{\sqrt{1-(5x^3)^2}}\cdot 15x^2$$



3. Let $\int_{-3}^{-1} g(x)\,dx = 8$ and $\int_0^1 g(x)\,dx = -2$

a) If g(x) is even, find $\int_1^3 g(x)\,dx$ Answer: 4

4. Evaluate $\int x\sqrt{x-5}\,dx$

$$\int x(x-5)^{1/2}dx \qquad \boxed{u=x-5} \quad dx=du \quad \frac{du}{dx}=1 \to x=u+5$$

$$\int x\cdot u^{1/2}du = \int (u+5)u^{1/2}du = \int u^{3/2}+5u^{1/2}du = \frac{2}{5}u^{5/2}+\frac{2}{3}(5)u^{3/2}$$

Answer: $\frac{2}{5}(x-5)^{5/2}+\frac{10}{3}(x-5)^{3/2}+C$

5. Evaluate $\int_0^4 x\sqrt{x^2+9}\,dx$

$$\int x(x^2+9)^{1/2}dx \qquad \int x\cdot u^{1/2}\cdot\frac{du}{2x}$$

Convert bounds
if $x=0$, $u=0+9=9$
if $x=4$, $u=16+9=25$

Answer: $\boxed{98/3}$

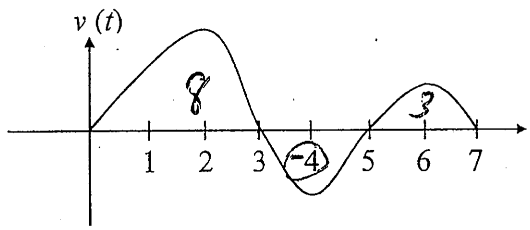
$u=x^2+9$

$\frac{du}{dx}=2x$ $\quad dx=\frac{du}{2x}$

$$\frac{1}{2}\int u^{1/2}du \qquad \left(\frac{2}{3}\cdot\frac{1}{2}\right)u^{3/2}\Big]_9^{25} = \frac{1}{3}u^{3/2}\Big]_9^{25} = \frac{1}{3}(25)^{3/2}-\frac{1}{3}(9)^{3/2}$$

$$=\frac{1}{3}(125)-\frac{1}{3}(27)=\frac{125-27}{3}$$

25pts



6. (2 points each part) A particle moves horizontally so that its velocity at time t, for $0 \le t \le 7$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0, 3, 5$ and 7 and the graph has horizontal tangents at $t = 2, 4,$ and 6.

The areas of the regions bounded by the t-axis and the graph of v on the intervals [0, 3], [3, 5] and [5, 7] are ~~8, 4, and 3~~ respectively. The position function for the particle is called x and at $t = 0$, $x(0) = 3$. 7, 5, 4

a. Find the positions of the particle at $t = 3$, $t = 5$ and $t = 7$. (show work using definite integrals.)

$x(3) = x(0) + \int_0^3 v(t)dt = 3 + 8 = 11$ 1

$x(5) = x(0) + \int_0^5 v(t)dt = 3 + 4 = \cancel{7}$ 6

$x(7) = x(0) + \int_0^7 v(t)dt = 3 + 7 = \cancel{10}$ 10

b. On what intervals (if any) is the acceleration negative? Justify your answer.

$v(t)$: + 0 3 − 5 + 7

$a(t)$: + 0 2 − 4 + 6 − 7

$(2,4) \cup (6,7)$ b/c $a(t) < 0$

c. On the interval $4 < t < 5$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

Speed is decreasing since $a(t)$ and $v(t)$ have opposite signs 1

d. Find the total distance traveled by the particle from $t = 0$ to $t = 7$. (Show proper integral notation)

$\int_0^7 |v(t)|dt = 8 + 4 + 3 = \boxed{15}$ 18

e. Find the total displacement of the particle from $t = 0$ to $t = 7$. (Show proper integral notation)

$\int_0^7 v(t)dt = 8 - 4 + 3 = \boxed{7}$ 8

10 pts

A.P. Calculus AB **Quiz 4-3, 4-4, 4-5**

NO CALCULATORS!

Justify your answers and show your work.

Name: Key B Date: 1/27/2017 (Fri) Per.

5 | 1. Money is withdrawn from an account in such a manner that the amount of money A (in thousands of dollars) in the account at time t years is given by the equation: $\frac{1}{9-0}\int_0^9 16-4t^{1/2}\,dt$

$$A(t) = 16 - 4\sqrt{t} \quad \text{for } t \geq 0.$$

At what time does the account have the average amount of money in it during the first 9 years.

$\frac{1}{9}\left(16t - \frac{2}{3}4t^{3/2}\right) = \frac{16}{9}(9) - \frac{8}{27}(9)^{3/2} - (0-0)$

$\left.\frac{16}{9}t - \frac{8}{27}t^{3/2}\right]_0^9 = 16 - \frac{8}{27}(27) = 16 - 8 = 8$

$= 8$

$16 - 4\sqrt{t} = 8$

$4\sqrt{t} = 8$ $\quad \sqrt{t} = 2 \quad t = 4$

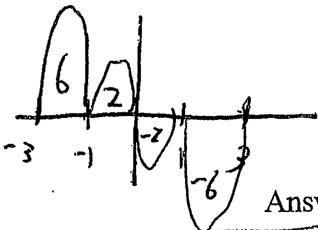
Answer: 4 yrs.

5 | 2. If $f(x) = \int_{-2x^3}^{-5} \frac{-2}{\sqrt{4-t^2}}\,dt$, find $\frac{d}{dx}f(x)$.

Answer: $\frac{-12x^2}{\sqrt{4-4x^6}} = \frac{-12x^2}{2\sqrt{1-x^6}} = \frac{-6x^2}{\sqrt{1-x^6}}$

$\frac{d}{dx}\left[\int_{-5}^{-2x^3} \frac{-2}{\sqrt{4-t^2}}\,dt\right] = \frac{+2}{\sqrt{4-(-2x^3)^2}} \cdot -6x^2 = \frac{-12x^2}{\sqrt{4-4x^6}}$

5 | 3. Let $\int_{-3}^{-1} g(x)\,dx = 6$ and $\int_0^1 g(x)\,dx = -2$



If g(x) is odd, find $\int_1^3 g(x)\,dx$

Answer: -6

5 | 4. Evaluate $\int x\sqrt{x-4}\,dx$

$\int x(x-4)^{1/2}dx$ $\quad u = x-4 \quad dx = du$

$\frac{du}{dx} = 1 \quad x = u+4$

$\int x \cdot u^{1/2}du$ | $\int (u+4)u^{1/2}du$ | $\int u^{3/2} + 4u^{1/2}du$ | $\frac{2}{5}u^{5/2} + \frac{2}{3}4u^{3/2}$ | $\frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + C$

Answer: $\frac{2}{5}(x-4)^{5/2} + \frac{8}{3}(x-4)^{3/2} + C$

5 | 5. Evaluate $\int_0^3 x\sqrt{x^2+16}\,dx$ $\quad \int x(x^2+16)^{1/2}dx$

Answer: ______

$u = x^2+16$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int x \cdot u^{1/2} \cdot \frac{du}{2x}$

$\frac{1}{2}\int u^{1/2}du$

$\frac{1}{2} \cdot \frac{2}{3}u^{3/2}$

$\left.\frac{1}{3}u^{3/2}\right]_{16}^{25} = \frac{1}{3}(25)^{3/2} - \frac{1}{3}(16)^{3/2}$

convert bounds:

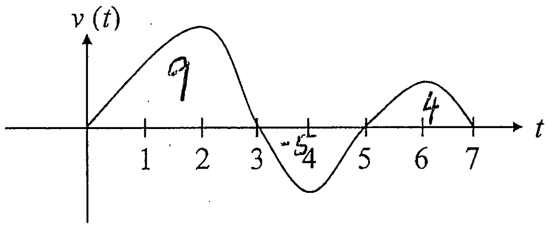
if $x=0$, $u=16$

if $x=3$, $u=25$

$\frac{1}{3}(125) - \frac{1}{3}(64)$

$\frac{125}{3} - \frac{64}{3} = \boxed{\frac{61}{3}}$

25



6. (each part 2 points) A particle moves horizontally so that its velocity at time t, for $0 \le t \le 7$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0, 3, 5$ and 7 and the graph has horizontal tangents at $t = 2, 4,$ and 6.

The areas of the regions bounded by the t-axis and the graph of v on the intervals [0, 3], [3, 5] and [5, 7] are 9, 5, and 4 respectively. The position function for the particle is called x and at $t = 0$, $x(0) = 2$.

2 a. Find the positions of the particle at $t = 3$, $t = 5$ and $t = 7$. (show work using definite integrals.)

$$x(3) = x(0) + \int_0^3 v(t)\,dt = 2 + 9 = 11$$

$$x(5) = x(0) + \int_0^5 v(t)\,dt = 2 + 4 = 6$$

$$x(7) = x(0) + \int_0^7 v(t)\,dt = 2 + 8 = 10$$

2 b. On what intervals (if any) is the acceleration positive? Justify your answer.

$v(t)$: + | − | + at 0, 3, 5, 7

$(0,2) \cup (4,6)$ b/c $a(t) > 0$

$a(t)$: + | − | + | − at 0, 2, 4, 6, 7

2 c. On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

increasing speed since $a(t)$ and $v(t)$ have same signs

2 d. Find the total distance traveled by the particle from $t = 3$ to $t = 7$. (Show proper integral notation)

$$\int_3^7 |v(t)|\,dt = 5 + 4 = \boxed{9}$$

2 e. Find the total displacement of the particle from $t = 3$ to $t = 7$. (Show proper integral notation)

$$\int_3^7 v(t)\,dt = -5 + 4 = \boxed{-1}$$

10

A.P. Calculus AB **Quiz 4-3, 4-4, 4-5**

NO CALCULATORS!

Justify your answers and show your work.

Each question worth 5 points unless otherwise specified.

Name Key C Date Per.

1. Money is withdrawn from an account in such a manner that the amount of money A (in thousands of dollars) in the account at time t years is given by the equation:

$$A(t) = 12 - 3\sqrt{t} \text{ for } t \geq 0.$$

At what time does the account have the average amount of money in it during the first 4 years.

$\frac{1}{4-0}\int_0^4 12 - 3t^{1/2}\,dt$ $\quad \frac{2}{3}\cdot 3t^{3/2}$

$\frac{1}{4}(32) = 8$

$8 = 12 - 3\sqrt{t}$

$-4 = -3\sqrt{t}$

$\frac{4}{3} = \sqrt{t}$

$\frac{16}{9} = t$

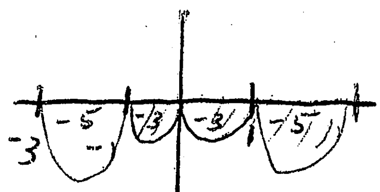
Answer: $16/9$ yr.

2. If $f(x) = \int_{2x^3}^{-3} \frac{3}{\sqrt{1-t^2}}\,dt$, find $\frac{d}{dx} f(x)$. Answer: ____________

$-\int_{-3}^{2x^3} \frac{3}{\sqrt{1-t^2}} = -\frac{3}{\sqrt{1-(2x^3)^2}} \cdot 6x^2$

$$\boxed{\frac{-18x^2}{\sqrt{1-4x^6}}}$$

3. Let $\int_{-1}^{-3} g(x)\,dx = 5$ and $\int_0^1 g(x)\,dx = -3$



If g(x) is even, find $\int_{-1}^{3} g(x)\,dx$ Answer: -11

4. Evaluate $\int 4x\sqrt{5-x}\,dx$

$u = 5 - x$ $\quad x = 5 - u$

$\frac{du}{dx} = -1$

$dx = -du$

Answer: $-\frac{40}{3}(5-x)^{3/2} + \frac{8}{5}(5-x)^{5/2} + C$

$\int 4x \cdot u^{1/2}(-du)$

$\int 4(5-u)u^{1/2}(-du)$

$\int -20u^{1/2} + 4u^{3/2}\,du = \frac{-20u^{3/2}}{3/2} + \frac{4u^{5/2}}{5/2} + C$

5. Evaluate $\int_0^4 x\sqrt{x^2+9}\,dx$ Answer: ____________

convert bounds

if $x = 0, u = 9$

if $x = 4, u = 25$

$u = x^2 + 9$

$\frac{du}{dx} = 2x$

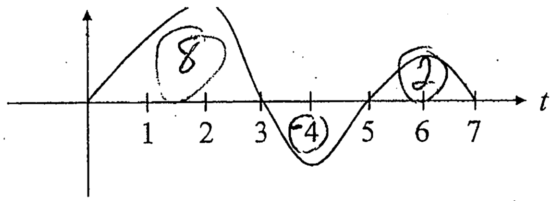
$dx = \frac{du}{2x}$

$\int x \cdot u^{1/2} \cdot \frac{du}{2x}$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right)$

$\frac{1}{3}u^{3/2}\Big]_9^{25} = \frac{1}{3}(25)^{3/2} - \frac{1}{3}(9)^{3/2} = \frac{125}{3} - \frac{27}{3} = \boxed{\frac{98}{3}}$



6. (each part 2 points) A particle moves horizontally so that its velocity at time t, for $0 \le t \le 7$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0, 3, 5$ and 7 and the graph has horizontal tangents at $t = 2, 4,$ and 6.

The areas of the regions bounded by the t-axis and the graph of v on the intervals [0, 3], [3, 5] and [5, 7] are 8, 4, and 2 respectively. The position function for the particle is called x and at $t = 0$, $x(0) = -2$.

2 a. Find the positions of the particle at $t = 3$, $t = 5$ and $t = 7$. (show work using definite integrals.)

$$x(3) = x(0) + \int_0^3 v(t)\,dt = -2 + 8 = 6$$

$$x(5) = x(0) + \int_0^5 v(t)\,dt = -2 + 4 = 2$$

$$x(7) = x(0) + \int_0^7 v(t)\,dt = -2 + 6 = 4$$

2 b. On what intervals (if any) is the acceleration negative? Justify your answer.

$a(t)$ is negative $(2,4) \cup (6,7)$ since $a(t) < 0$
$(v'(t) < 0)$

2 c. On the interval $4 < t < 5$, is the velocity of the particle increasing or decreasing? Give a reason for your answer.

$v(t)$ is increasing since $a(t) > 0$

2 d. Find the total distance traveled by the particle from $t = 3$ to $t = 7$. (Show proper integral notation)

$$\int_3^7 |v(t)|\,dt = 4 + 2 = \boxed{6}$$

2 e. Find the total displacement of the particle from $t = 3$ to $t = 7$. (Show proper integral notation)

$$\int_3^7 v(t)\,dt = -4 + 2 = \boxed{-2}$$

A.P. Calculus AB Quiz 4-3, 4-4, 4-5

NO CALCULATORS! Justify your answers.

Name Key D Period

1. A company introduces a new product, and the monthly profit over the first 9 months is approximated by the model $P(t) = 9(\sqrt{t} + 30)$, where P is the monthly profit in thousands of dollars and t is the time in months ($t = 0$ to $t = 9$). Find the average monthly profit for this timeframe. (and time)

Answer: \$288

$\frac{2592}{9} = 288$

$P(t) = \frac{1}{9-0}\int_0^9 9(\sqrt{t}+30)\,dt$ $\int 9t^{1/2} + 270\,dt$

$288 = 9(\sqrt{t}+30)$
$32 = \sqrt{t} + 30$
$2 = \sqrt{t}$
$\boxed{4 = t}$

$\frac{1}{9}\left[\frac{9t^{3/2}}{3/2} + 270t\right] = \frac{2}{3}t^{3/2} + 30t\Big]_0^9 = \frac{2}{3}(9)^{3/2} + 30(9) =$

$\frac{1}{9}\cdot\frac{2}{3}9t^{3/2} + \frac{270}{9}t$

$= \frac{2}{3}(27) + 270 = 270 + 18 = 288$

2. If $f(x) = \int_{2x^3}^{5} \frac{1}{\sqrt{1-t^2}}\,dt$ find $\frac{d}{dx}f(x)$.

Answer: $-\frac{1}{\sqrt{1-(2x^3)^2}}\cdot 6x^2 = \boxed{\frac{-6x^2}{\sqrt{1-4x^6}}}$

3. Evaluate $\int x\sec^2(5x^2)\,dx$

Answer: $\frac{1}{10}\tan(5x^2) + C$

$u = 5x^2$
$\frac{du}{dx} = 10x$
$dx = \frac{du}{10x}$

$\int x \cdot \sec^2 u \cdot \frac{du}{10x}$

$\frac{1}{10}\int \sec^2 u\,du =$

$\frac{1}{10}\tan u + C$

4. Evaluate $\int x\sqrt{9-x}\,dx$

Answer: $-6(9-x)^{3/2} + \frac{2}{5}(9-x)^{5/2} + C$

$u = 9-x$
$dx = -du$ $\frac{du}{dx} = -1$
$x = 9-u$

$\int (9-u)u^{1/2}(-du)$
$\int -u^{1/2}(9-u)\,du$

$\int -9u^{1/2} + u^{3/2}\,du$
$-9\cdot\frac{u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2}$

$-9\cdot\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2}$

5. Evaluate $\int_0^3 \frac{2x}{\sqrt{25-x^2}}\,dx$

Answer: $+2$

if $x = 0,\ u = 25$
If $x = 3,\ u = 25 - 9 = 16$

$u = 25 - x^2$
$\frac{du}{dx} = -2x$

$\int \frac{2x}{u^{1/2}}\cdot\frac{du}{-2x}$

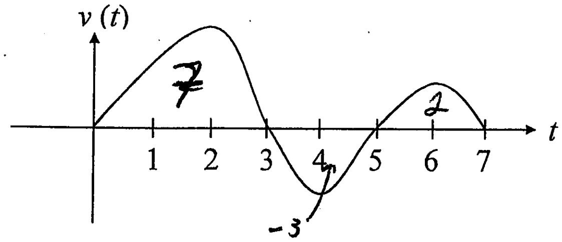
$-\int u^{-1/2}\,du$

$-\frac{u^{1/2}}{1/2}$

$-2u^{1/2}\Big]_{25}^{16}$

$-2\sqrt{16} - (-2\sqrt{25})$

$-2(4) + 2(5)$
$= -8 + 10 = 2$



6. A particle moves horizontally so that its velocity at time t, for $0 \le t \le 7$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0, 3, 5$ and 7 and the graph has horizontal tangents at $t = 2, 4$, and 6.

The areas of the regions bounded by the t-axis and the graph of v on the intervals [0, 3], [3, 5] and [5, 7] are 7, 3, and 2 respectively. The position function for the particle is called x and at $t = 0$, $x(0) = 5$.

a. Find the positions of the particle at $t = 3$, $t = 5$ and $t = 7$. (Hint: use definite integrals.)

$x(3) = x(0) + \int_0^3 v(t)\,dt = 5 + 7 = 12$

$x(5) = x(0) + \int_0^5 v(t)\,dt = 5 + 7 - 3 = 9$

$x(7) = x(0) + \int_0^7 v(t)\,dt = 5 + 7 - 3 + 2 = 11$

b. On what intervals (if any) is the acceleration negative? Justify your answer.

$(2,4) \cup (6,7)$ b/c $a(t) < 0$

c. On the interval $4 < t < 5$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

Dec, $a(t), v(t)$ opp. signs

d. Find the total distance traveled by the particle from $t = 0$ to $t = 7$. (use integral notation)

$\int_0^7 |v(t)|\,dt = 12$

e. Find the total displacement of the particle from $t = 3$ to $t = 7$. (use integral notation)

$\int_3^7 v(t)\,dt = -1$