A.P. Calculus AB **4.3 – 4.5 (Morning Quiz Review Session)**

NO CALCULATORS!

1. Given f(x) = $\frac{4(x^2+1)}{x^2}$, find a) average value in the interval [1, 3] b) find the value of c guaranteed by the theorem

2. If $f(x) = \int_{3x^4}^{-5} \frac{5}{2\sqrt{t}}\,dt$, find $\frac{d}{dx}f(x)$.

3. If $f(x) = \int_{-2x^2}^{\sqrt{x}} t^2 - 2t\,dt$, find $\frac{d}{dx}f(x)$.

4. Let $\int_{-3}^{6} g(x)\,dx = -4$ and $\int_{0}^{3} g(x)\,dx = 2$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$

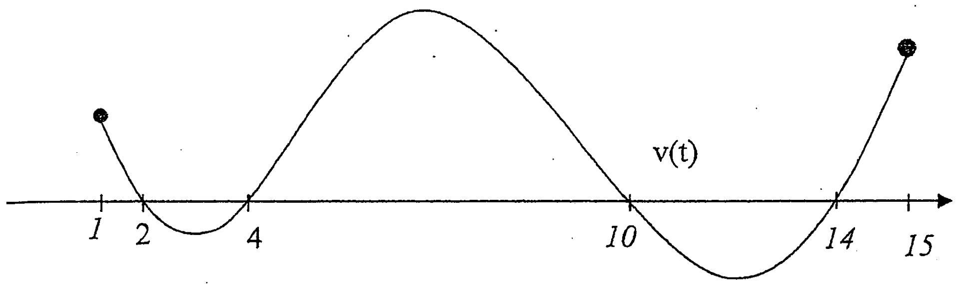
5. Evaluate $\int \sqrt{\tan x}\sec^2 x dx$

6. Evaluate $\int 2x\sqrt{4-x}\,dx$

7. Evaluate $\int_{0}^{1} x^2(x^3+1)^3\,dx$

AP Calculus AB (4.3-4.5) morning review

Velocity graph particle motion problem



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 15$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 4, 10 and 14 and the graph has horizontal tangents at $t = 3$, 7, and 12.

The areas of the regions bounded are **1, 2, 14, 6, and 3** respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 3$.

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $10 < t < 12$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 2$, $t = 4$ and $t = 10$, $t = 14$. (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

h. Find the total distance traveled by the particle from $t = 1$ to $t = 15$. (Use Integral Notation)

i. Find the total displacement of the particle from $t = 2$ to $t = 15$. (Use Integral Notation)

A.P. Calculus AB — 4.3 – 4.5 (Morning Quiz Review Session)

NO CALCULATORS!

1. Given $f(x) = \frac{4(x^2+1)}{x^2}$, find a) average value in the interval [1, 3] b) find the value of c guaranteed by the theorem

$f(x) = \frac{4x^2+4}{x^2} = (4x^2+4)x^{-2} = 4+4x^{-2}$

Avg. value $= \frac{1}{3-1}\int_1^3 4+4x^{-2}\,dx$

$= \frac{1}{2}\cdot 4x + \frac{4x^{-1}}{-1}\Big]_1^3$

$\frac{1}{2}\cdot\left[12-\frac{4}{3}-(4-4)\right]$

$= \frac{1}{2}\left[\frac{36}{3}-\frac{4}{3}\right] = \frac{1}{2}\left(\frac{32}{3}\right) = \boxed{\frac{16}{3}}$

b) set $f(x) = \frac{16}{3}$

$4+\frac{4}{x^2} = \frac{16}{3}$

$\frac{4}{x^2} = \frac{16}{3} - \frac{4}{1}$

$\frac{4}{x^2} = \frac{4}{3}$

$x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{c=\sqrt{3}}$

2. If $f(x) = \int_{3x^4}^{-5} \frac{5}{2\sqrt{t}}\,dt$, find $\frac{d}{dx}f(x)$. $\boxed{\text{SFTC}}$

$f(x) = \int_{-5}^{3x^4} -\frac{5}{2\sqrt{t}} \rightarrow \frac{d}{dx}\int_{-5}^{3x^4}\frac{-5}{2\sqrt{t}} = \frac{-5}{2\sqrt{3x^4}}\cdot 12x^3 = \frac{-30x^3}{\sqrt{3}x^2}$

$= \boxed{\frac{-30x}{\sqrt{3}}}$

3. If $f(x) = \int_{-2x^2}^{\sqrt{x}} t^2-2t\,dt$, find $\frac{d}{dx}f(x)$. $\boxed{\text{SFTC}}$

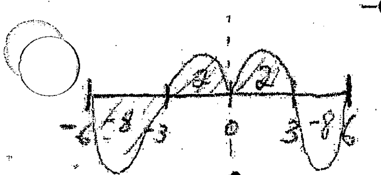
$(x-2\sqrt{x})\cdot\frac{1}{2}x^{-1/2} - \left[4x^4+4x^2\right](-4x)$

$(x-2\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) + 16x^5 + 16x^3$

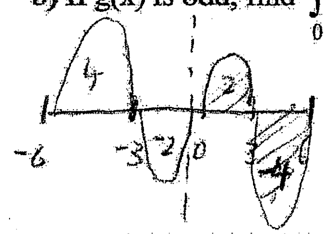
$\boxed{\frac{\sqrt{x}}{2} - 1 + 16x^5 + 16x^3}$

4. Let $\int_{-3}^{6} g(x)\,dx = -4$ and $\int_{0}^{3} g(x)\,dx = 2$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx = \boxed{-4}$



b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx = \boxed{-2}$



5. Evaluate $\int \sqrt{\tan x}\sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int u^{1/2}\cdot\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}} = \frac{u^{3/2}}{3/2} + C = \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

6. Evaluate $\int 2x\sqrt{4-x}\,dx$ $x = 4-u$

$u = 4-x$

$\frac{du}{dx} = -1$

$dx = -du$

$\int 2x\cdot u^{1/2}\cdot(-du)$

$\int 2(4-u)(u^{1/2})(-1)\,du$

$\int -2u^{1/2}(4-u)\,du$

$\int -8u^{1/2} + 2u^{3/2}\,du$

$= -8\frac{u^{3/2}}{3/2} + 2\frac{u^{5/2}}{5/2} + C$

$= -8\left(\frac{2}{3}\right)u^{3/2} + 2\left(\frac{2}{5}\right)u^{5/2} + C$

$= -\frac{16}{3}(4-x)^{3/2} + \frac{4}{5}(4-x)^{5/2} + C$

7. Evaluate $\int_0^1 x^2(x^3+1)^3\,dx$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int x^2\cdot u^3\cdot\frac{du}{3x^2} = \frac{1}{3}\int u^3\,du$

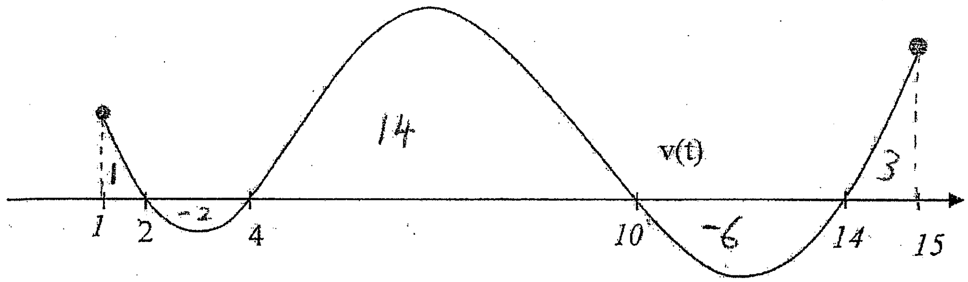
= if $x=0$, $u = 0^3+1 = 1$

if $x=1$, $u = 1^3+1 = 2$

$\frac{1}{3}\cdot\frac{u^4}{4}\Big]_1^2 = \frac{1}{3}\left(\frac{2^4}{4} - \frac{1^4}{4}\right)$

$= \frac{1}{3}\left(\frac{15}{4}\right) = \frac{15}{12} = \boxed{\frac{5}{4}}$

Velocity graph particle motion problem



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 15$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 4, 10 and 14 and the graph has horizontal tangents at $t = 3$, 7, and 12.

The areas of the regions bounded are **1, 2, 14, 6, and 3** respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 3$.

a. Create Sign lines for v(t) and a(t)

$v(t)$: + on (1, 2), − on (2, 4), + on (4, 10), − on (10, 14), + on (14, 15); marks 1, 2, 4, 10, 14, 15

$a(t)$: − on (1, 3), + on (3, 7), − on (7, 12), + on (12, 15); marks 1, 3, 7, 12, 15

b. On what intervals (if any) is the velocity negative? Justify your answer.

$(2,4) \cup (10,14)$ b/c $v(t) < 0$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$(3,7) \cup (12,15)$ b/c $a(t) > 0$

d. On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer. decreasing speed b/c $v(t) < 0$, $a(t) > 0$, (opposite signs)

e. On the interval $10 < t < 12$, is the speed of the particle increasing or decreasing? Give a reason for your answer. increasing speed b/c $a(t) < 0$, $v(t) < 0$, (same signs)

f. Find the positions of the particle at $t = 2$, $t = 4$ and $t = 10$, t = 14. (use definite integrals.)

$x(2) = x(1) + \int_1^2 v(t)\,dt = 3 + 1 = 4$

$x(10) = x(2) + \int_2^{10} v(t)\,dt = 4 + -2 + 14 = 16$

$x(14) = x(10) + \int_{10}^{14} v(t)\,dt = 16 - 6 = 10$

g. State the absolute extrema and the t-values where they occur.

Test other critical pt and endpt.

$x(4) = x(2) + \int_2^4 v(t)\,dt = 4 - 2 = 2$

$x(15) = x(14) + \int_{14}^{15} v(t)\,dt = 10 + 3 = 13$

$\boxed{\text{Abs. min is } 2 \text{ at } t=4 \quad \text{Abs. max is } 16 \text{ at } t=10}$

h. Find the total distance traveled by the particle from $t = 1$ to $t = 15$. (Use Integral Notation)

$\int_1^{15} |v(t)|\,dt = 1 + 2 + 14 + 6 + 3 = \boxed{26}$

i. Find the total displacement of the particle from $t = 2$ to $t = 15$. (Use Integral Notation)

$\int_2^{15} v(t)\,dt = -2 + 14 - 6 + 3 = \boxed{9}$