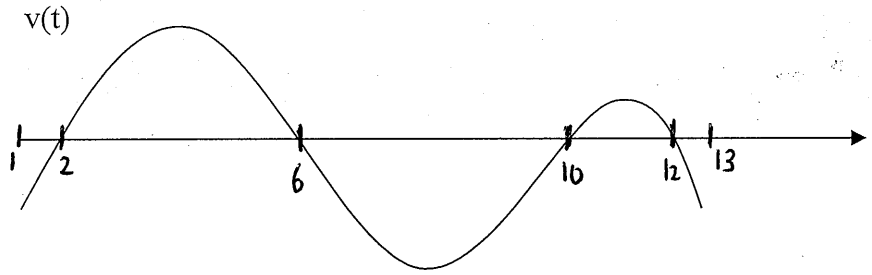
AP Calculus AB (4.3-4.5) Quiz Review : PVA Particle Motion Problem #2:



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 13$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 2$, 6, 10 and 12 and the graph has horizontal tangents at $t = 4$, 8, and 11.

The areas of the regions bounded are 1,8, 10, 3, 2 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = -3$

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $8 < t < 10$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $12 < t < 13$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 2$, $t = 6$ and $t = 10$, and t = 12 (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

h. Find the total distance traveled by the particle from $t = 1$ to $t = 13$. (Use Integral Notation)

i. Find the total displacement of the particle from $t = 6$ to $t = 13$. (Use Integral Notation)

j. Sketch graph of x(t) below:

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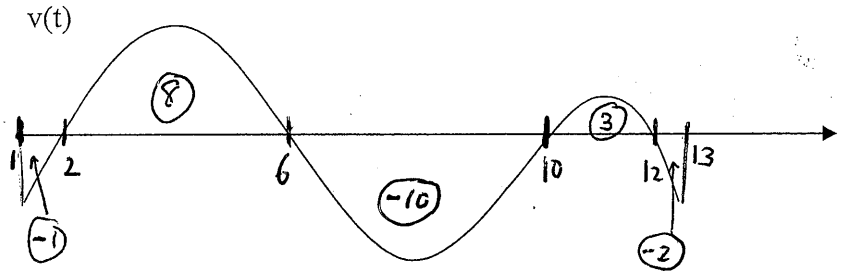
Key



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a. Create Sign lines for v(t) and a(t)

v(t) − + − + −
1 2 6 10 12 13

a(t) + − + −
1 4 8 11 13

b. On what intervals (if any) is the velocity negative? Justify your answer.

$v(t) < 0$ on $(1,2) \cup (6,10) \cup (12,13)$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$a(t) > 0$ on $(1,4) \cup (8,11)$

d. On the interval $8 < t < 10$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

speed is decreasing since $v(t) < 0$, $a(t) > 0$ (opposite signs)

e. On the interval $12 < t < 13$ is the speed of the particle increasing or decreasing? Give a reason for your answer.

speed is increasing since $a(t) < 0$, $v(t) < 0$ (same signs)

final position = given position + displacement

$* x(b) = x(a) + \int_a^b v(t)\,dt$

f. Find the positions of the particle at $t = 2$, $t = 6$ and $t = 10$, and t = 12 (use definite integrals.)

$x(2) = x(1) + \int_1^2 v(t)\,dt$

$x(2) = -3 + (-1) = \boxed{-4}$

$x(6) = x(1) + \int_1^6 v(t)\,dt$

$= -3 + (-1+8) = \boxed{4}$

$x(10) = x(1) + \int_1^{10} v(t)\,dt$

$= -3 + (-1+8-10) =$

$= \boxed{-6}$

$x(12) = x(1) + \int_1^{12} v(t)\,dt$

$= -3 + (-1+8-10+3) = \boxed{-3}$

g. State the absolute extrema and the t-values where they occur.

Abs max value is 4 at $x = 6$

Abs min value is -4 at $x = 2$

h. Find the total distance traveled by the particle from $t = 1$ to $t = 13$. (Use Integral Notation)

$\int_1^{13} |v(t)|\,dt = \boxed{24}$

i. Find the total displacement of the particle from $t = 6$ to $t = 13$. (Use Integral Notation)

$\int_6^{13} v(t)\,dt = -10 + 3 - 2 = \boxed{-9}$

j. Sketch graph of x(t) below: