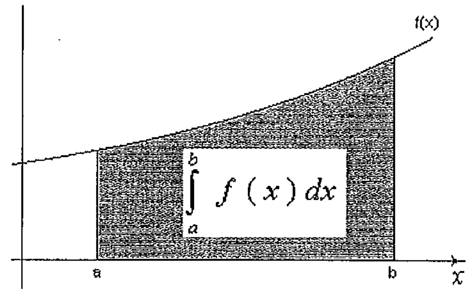
Calculus **Chapters 4.3/4.4a** **1st Fundamental Theorem of Calculus**

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is the antiderivative of f.



Recall:

*The general derivative is a **slope-finding function** or formula : (ex. f '(x) = 2x + 1)

*The specific derivative is the **actual slope** at a point (ex: f '(3) = 7)

Likewise…

The indefinite integral is an **Area-Finding Function** or formula (Ex: $\int 2x dx = x^2 + C$)

The definite integral is the **Actual Area** of the region for an interval (Ex: $\int_1^3 2x dx = 8$)

*If a function is **continuous** on a closed interval, then the function is able to be integrated on that interval

Class Examples:

1. Evaluate $\int_1^4 (3x^2 + 4x - 1)dx$

**NOTE: For definite integrals, we don't need to worry about the constant of integration " +C ". It will always wash out.

2. Evaluate $\int_{-2}^1 2x dx$

②

Integral Properties:

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (given that c is between a and b)

Example 3: If $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, find the below:

a) $\int_0^6 f(x)dx$

b) $\int_6^3 f(x)dx$

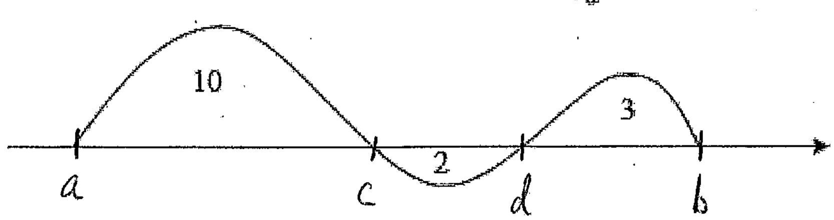
c) $\int_3^3 f(x)dx$

d) $\int_3^6 (-5f(x)+3)dx$

Ex. 4: If $\int_3^8 f'(x)dx = 10$ and $f(8) = 6$, find $f(3)$.

*Reminder that the FFTC can be used as an equation solving tool to find the value of an antiderivative at a specific point. Recall that $\int_a^b f'(x)dx = f(b) - f(a)$

Ex. 5: The area for each region is given. Find $\int_a^b f(x)dx$

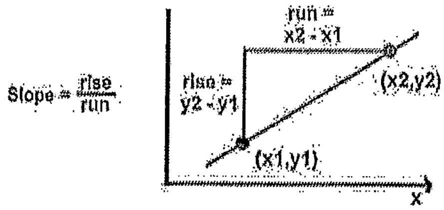


Differential Calculus vs Integral Calculus Summary Sheet

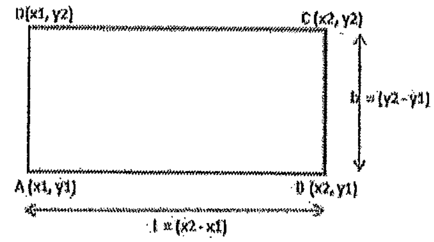
Differential Calculus (Derivative)
Explores rates of change

Integral Calculus (Antiderivative)
Explores the accumulation of change

$$Slope\ m = \frac{y_2 - y_1}{x_2 - x_1}$$

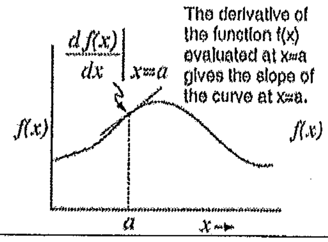


$$Area = (x_2 - x_1) \times (y_2 - y_1)$$



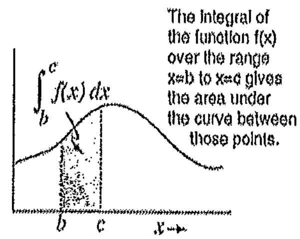
Derivative

$$\frac{d\,f(x)}{dx}$$



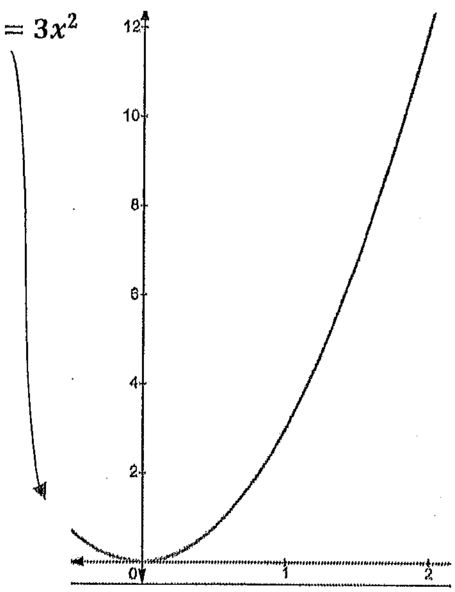
Integral

$$\int f(x)\,dx$$

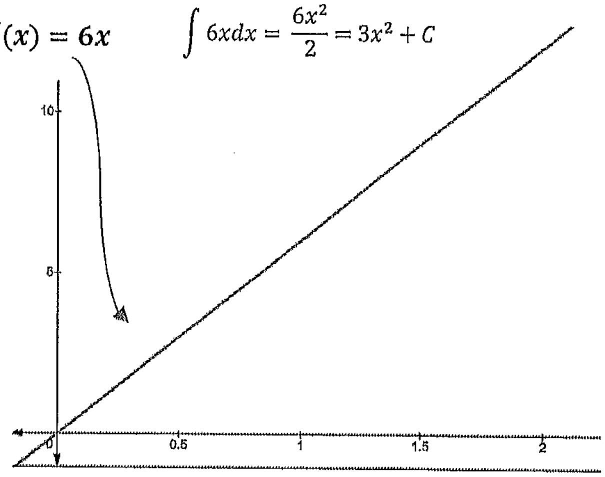


$$f(x) = 3x^2$$

$$\frac{d}{dx}3x^2 = 6x$$



$$f'(x) = 6x \qquad \int 6x dx = \frac{6x^2}{2} = 3x^2 + C$$



* *The* ***area under*** *the derivative graph is equal to the* ***rise in height*** *of the antiderivative graph*

* *The average slope of the antiderivative graph is equal to the average height of region under derivative graph*

$$Avg\ value\ f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

Second Fundamental Theorem of Calculus (SFTC)

$$\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) * p'(x)$$

First Fundamental Theorem of Calculus (FFTC)

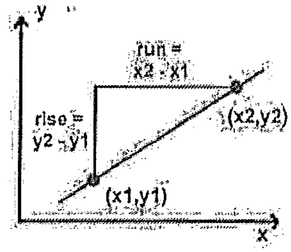
$$\int_a^b f'(x)dx = f(a) - f(b)$$

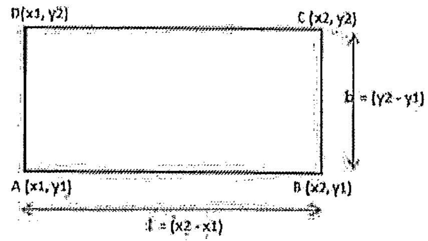
(4) Key

Differential Calculus vs Integral Calculus Summary Sheet

Differential Calculus (Derivative)	Integral Calculus (Antiderivative)
$Slope\ m = \frac{y_2 - y_1}{x_2 - x_1}$	$Area = (x_2 - x_1) \times (y_2 - y_1)$

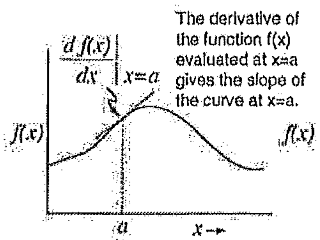
Slope = $\frac{rise}{run}$





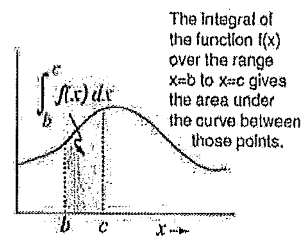
Derivative

$$\frac{d\,f(x)}{dx}$$



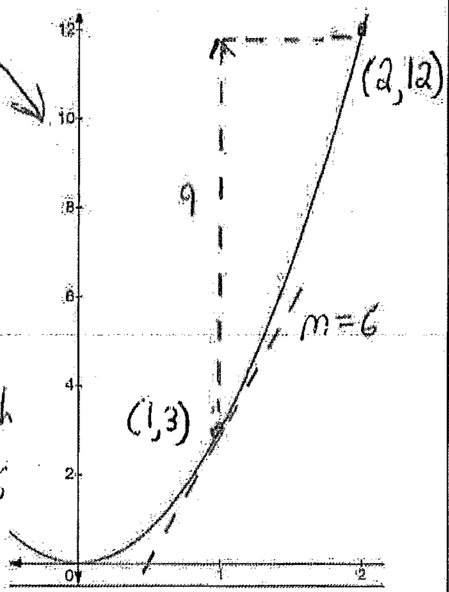
Integral

$$\int f(x)\,dx$$



$f(x) = 3x^2$

$\frac{d}{dx}3x^2 = 6x$

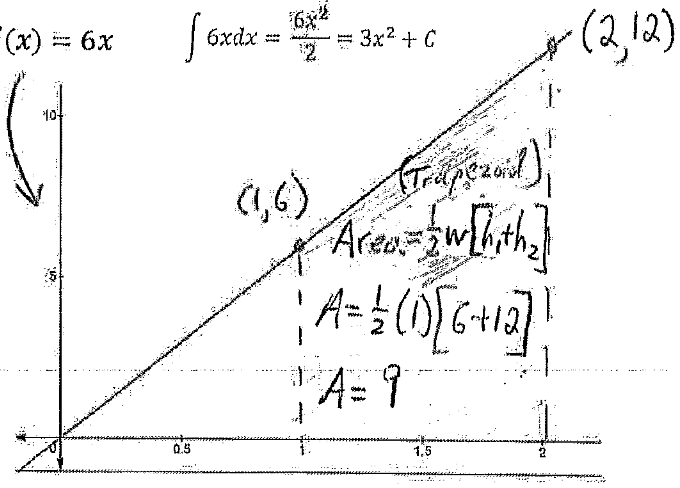


• The derivative function can find the slope of any point on this graph

ex: $f'(1) = 6(1) = 6$

$f(2) - f(1) = 12 - 3 = \boxed{9}$

$f'(x) = 6x$ $\quad \int 6x\,dx = \frac{6x^2}{2} = 3x^2 + C$



• The Integral of this function can be used to find the area under this graph

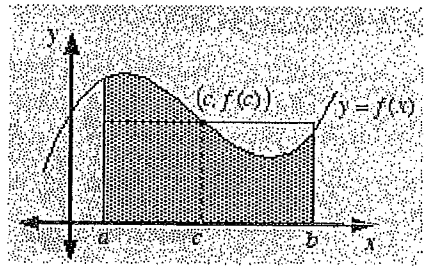
$$\int_1^2 6x\,dx = \frac{6x^2}{2} \rightarrow 3x^2\Big]_1^2 = 3(2)^2 - 3(1)^2 = \boxed{9}$$

Second Fundamental Theorem of Calculus (SFTC)	First Fundamental Theorem of Calculus (FFTC)
$\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) * p'(x)$	$\int_a^b f'(x)dx = f(a) - f(b)$

Calculus **Chapter 4.4b** **Average Value Theorem (MVT for Integrals)**

If function f is integrable on the closed interval [a, b], then the average value of f on the interval is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$



*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

Example 1: a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

(6)

2nd Fundamental Theorem of Calculus (SFTC)

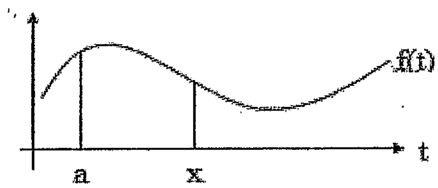
Definite Integral as a Function

To recap, we've covered:

1) Indefinite General Integrals (Area-finding functions)
2) Definite Integrals (Finds Area between 2 x-values)

There is also now a function that is the integral itself. Instead of going from a constant to another constant, we are going from a constant to a moving value of x.

Consider: $f(x)=\int_a^x f(t)dt$



2nd Fundamental Theorem of Calculus **Very Important**

Applies the concept that derivative and integrals are inverse operations of each other.

1) $\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)$ (a is a constant)

2) $\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)-f(q(x))\cdot q'(x)$

Example 2:

a) $\frac{d}{dx}\left[\int_{-3}^{x}\sqrt{t^2+4}dt\right]=$

b) $\frac{d}{dx}\left[\int_{3}^{x^2}\sqrt{t-1}dt\right]=$

c) $\frac{d}{dx}\left[\int_{10}^{x^2}\sqrt{t-1}dt\right]=$

d) $\frac{d}{dx}\left[\int_{3x}^{0}\frac{1}{t+2}dt\right]=$

e) $\frac{d}{dx}\left[\int_{x}^{x^2}(2t+3)dt\right]=$

Calculus **Chapter 4.5a** **U-Substitution Method for Indefinite Integrals**

U-Substitution is the reverse of the derivative chain rule. We want to look for a function and its derivative to be in the integral.

Example 1:

Suppose $f(x) = \sin(3x)$

$f'(x) = \cos(3x) \cdot 3$ $f'(x) = 3\cos(3x)$	This means that: $\int 3\cos(3x)dx = \sin(3x) + C$

U-Substitution Steps:

1. Assign the 'u' value to the expression inside the parentheses.
2. Find the derivative of u: $\frac{du}{dx}$
3. Solve for dx.
4. Rewrite the integral in terms of u and du. (Check to make sure no 'x' or 'dx' remains)
5. Evaluate the Integral
6. Write the answer in terms of x.
7. **Not all Integral problems require U-Substitution. Check first to see if expansion/rewriting problem will allow problem to only need the Power Rule**

Ex. 2: $\int x(x^2+1)^{15}dx$

Ex. 3: $\int x^2 \sec^2(2x^3)dx$

Ex. 4: $\int x^3\sqrt{5-x^4}\,dx$

Ex. 5: $\int \tan^5 x \sec^2 x \, dx$

Ex. 6: $\int (3-y)\left(\frac{1}{\sqrt{y}}\right) dy$

Change of Variable U-Substitution Method:

Ex. 7: $\int x\sqrt{x+3}\,dx$

Ex. 8: $\int x^2\sqrt{2-x}\,dx$

Calculus **Chapter 4.5b** **U-Substitution Method for Definite Integrals**

U-Substitution with definite integrals: Be sure the bounds matches the variable of the problem

Ex. 1: $\int_{1}^{2} 2x(x^2-2)^3\,dx$

Ex. 2: $\int_{1}^{5} \frac{x}{\sqrt{2x-1}}\,dx$

(10)

Integrals of Odd and Even Functions

Review: Suppose $\int_{10}^{3} f(x)dx = 9$ and $\int_{-1}^{3} f(x)dx = 5$, find $\int_{-1}^{10} f(x)dx$

Even/Odd Rules:

Even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Odd: $\int_{-a}^{a} f(x)dx = 0$

Ex. 3: Suppose g(x) is an even function where $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

(Sketch a possible graph using the above given information)

Ex. 4: Same as Example 3, but g(x) is an odd function: $\int_{0}^{3} g(x)dx = 2$ and $\int_{-4}^{-3} g(x)dx = 4$. Find $\int_{-4}^{3} g(x)dx$.

Ex. 5: If f(x) is even and $\int_{3}^{6} f(x)dx = 7$ and $\int_{-6}^{3} f(x)dx = 12$, find $\int_{0}^{6} f(x)dx$

Non-AP Calculus Chapter 4.5 Integration and U-Substitution Worksheet

1)

$$\int (5x+4)^5 \, dx$$

2)

$$\int 3t^2(t^3+4)^5 \, dt$$

3)

$$\int \sqrt{4x-5} \, dx$$

4)

$$\int \frac{5x^2}{\sqrt[5]{x^3-2}} dx$$

5)

$$\int \cos(2x+1) \, dx$$

6)

$$\int \sin^{10}(x)\cos(x) \, dx$$

(12)

7. $\int \frac{\sin(x)}{(\cos(x))^5}\,dx$

8) $\int \frac{2}{\sqrt{3x-7}}\,dx$

9) $\int \frac{4}{x^2}\sec\left(\frac{5}{x}\right)\tan\left(\frac{5}{x}\right)dx$

10) $\int \frac{3x^4}{\left(7-x^5\right)^6}\,dx$

11) $\int \frac{x^3(2x-1)}{\sqrt{x}}\,dx$

12) $\int 7x^2\sqrt{3-2x^3}\,dx$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$
29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$
30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\,du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$