1/2

4.3/4.4a (continued)

Integral Properties

1) $\int_a^a f(x)\,dx = 0$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(given that c is between a and b)

Ex. 1 If $\int_0^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = -1$

find:

a) $\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx$

$= 4 + -1 = \boxed{3}$

b) $\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = \boxed{1}$

c) $\int_3^3 f(x)\,dx = \boxed{0}$

d) $\int_3^6 -5f(x) + 3\,dx$

$= -5\int_3^6 f(x)\,dx + \int_3^6 3\,dx$

$3x\Big]_3^6 = 3(6) - 3(3) = 9$

$= -5(-1) + 9 = \boxed{14}$

* Be careful with these types of problems. The integral of 3 is not 3.

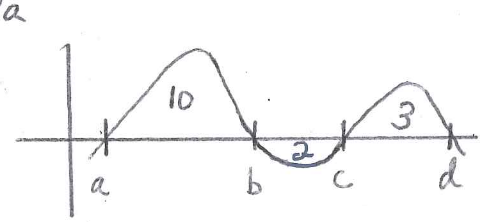
Ex. 2 If $\int_3^8 f'(x)\,dx = 10$ and $f(8) = 6$, find $f(3)$.

Using FFTC, $\int_3^8 f'(x)\,dx = f(8) - f(3)$

$10 = 6 - f(3) \rightarrow 4 = -f(3)$ so $\boxed{f(3) = -4}$

Ex. 3 The area for each region is shown below.

Find $\int_a^d f(x)\,dx$



$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$

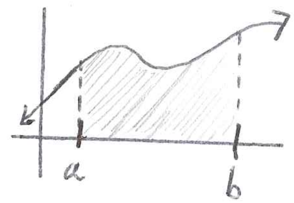
$= 10 + (-2) + 3$

$= \boxed{11}$

2/2

4.3/4.4a 1st Fundamental Theorem of Calculus and PVA

1st Fundamental Theorem of Calculus (FFTC)



$$\int_a^b f(x)\,dx = F(b) - F(a)$$ where F is the antiderivative of f.

Recall:

- The general derivative is a slope-finding function (ex. $f'(x) = 2x+1$)
- The specific derivative is the actual slope at a point (ex. $f'(3) = 7$)

Likewise...

- The indefinite integral is an area-finding function
- The definite integral is the actual area of region for an interval

* If a function is continuous on a closed interval, then the function is able to be integrated on that interval.

Ex. 1 Evaluate $\int_1^4 (3x^2+4x-1)\,dx$

* No need to worry about "+C" It will just naturally cancel out.

$$= \frac{3x^3}{3} + \frac{4x^2}{2} - x \Big]_1^4 \rightarrow x^3 + 2x^2 - x \Big]_1^4 = 4^3 + 2(4)^2 - 4 - \left(1^3 + 2(1)^2 - 1\right)$$

$$= 92 - 2 = \boxed{90}$$

Ex. 2 Evaluate $\int_{-2}^{1} 2x\,dx$

$$= \frac{2x^2}{2}\Big]_{-2}^{1} = x^2\Big]_{-2}^{1} = 1^2 - \left((-2)^2\right) = 1 - 4 = \boxed{-3}$$

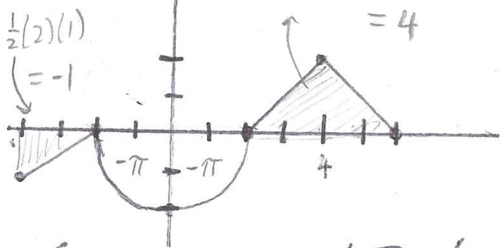
* portions of graph below x-axis will result in negative value

4.3-4.4a p.278-280 13-21 odd, 33-39 odd, 47, 49
p. 291-292 5-31 odd

$\boxed{4.3}$ p.280

47) $\frac{1}{2}(2)(1) = -1$

$\frac{1}{2}(b)h = \frac{1}{2}(4)(2) = 4$



a) $\int_0^2 f(x)dx = A_{circle} = \pi r^2$ Area of quarter-circle

$A_{quartercircle} = \frac{1}{4}(\pi r^2) = \frac{1}{4}\pi(2)^2 = \frac{4}{4}\pi = \pi = \boxed{-\pi}$

b) $\int_2^6 f(x)dx = \boxed{4}$ Area of Triangle

c) $\int_{-4}^2 f(x)dx = \boxed{-1-2\pi}$

d) $\int_{-4}^6 f(x)dx = -1-2\pi+4 = \boxed{3-2\pi}$

e) $\int_{-4}^6 |f(x)|dx = 1+2\pi+4 = \boxed{5+2\pi}$

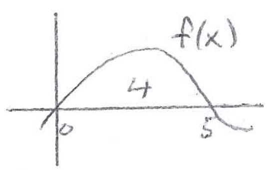
f) $\int_{-4}^6 [f(x)+2]dx = \int_{-4}^6 f(x)dx + \int_{-4}^6 2dx$ $\quad 2x\Big]_{-4}^6 = 12-(2(-4)) = 20$

careful with this calculation!

$-1-2\pi+4$
$= 3-2\pi$

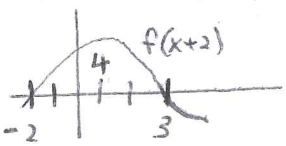
$= 3-2\pi+20$
$= \boxed{23-2\pi}$

49) $\int_0^5 f(x)dx = 4$



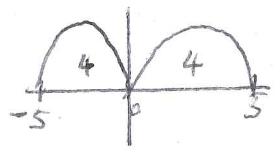
a) $\int_0^5 [f(x)+2] = \int_0^5 f(x)dx + \int_0^5 2dx = 4 + 2x\Big]_0^5 =$ $10-0=10$ $\quad 4+10 = \boxed{14}$

b) $\int_{-2}^3 f(x+2)dx$



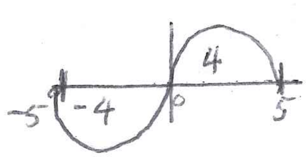
$= \boxed{4}$

c) $\int_{-5}^5 f(x)dx$ (f is even)



$= \boxed{8}$

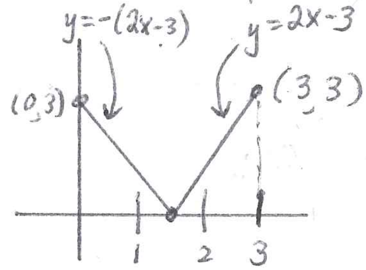
d) $\int_{-5}^5 f(x)dx$ (f is odd)



$= \boxed{0}$

4.4a p. 291-292 5-31 odd

23) $\int_0^3 |2x-3|\,dx$



Geometric method: *Add Areas of triangles $(A=\frac{1}{2}bh)$,

$$\frac{1}{2}\left(\frac{3}{2}\right)(3)+\frac{1}{2}\left(\frac{3}{2}\right)(3)=\frac{9}{4}+\frac{9}{4}=\frac{18}{4}=\boxed{\frac{9}{2}}$$

Integral method:

$$\int_0^{1.5}-(2x-3)\,dx+\int_{1.5}^{3}(2x-3)\,dx$$

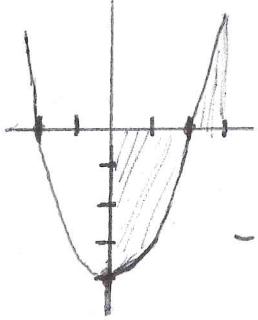
$$\left.\frac{2x^2}{2}-3x\right]_{1.5}^{3}=(9-9)-\left(1.5^2-3(1.5)\right)$$

$$=-(-2.25)=\underline{2.25}$$

$$\left.-\frac{\cancel{2}x^2}{\cancel{2}}+3x\right]_0^{1.5}=-(1.5)^2+3(1.5)=\underline{2.25}$$

$$2.25+2.25=4.5=\boxed{\frac{9}{2}}$$

25) $\int_0^3 |x^2-4|\,dx$



$$-\left[\int_0^2 x^2-4\,dx\right]+\int_2^3 x^2-4\,dx$$

$$-\left.\frac{x^3}{3}-4x\right]_0^2$$

$$-\left(\frac{8}{3}-8\right)-0=\underline{\underline{\frac{16}{3}}}$$

$$\left.\frac{x^3}{3}-4x\right]_2^3$$

$$\frac{27}{3}-12-\left(\frac{8}{3}-8\right)$$

$$=\underline{\underline{\frac{7}{3}}}$$

$$\frac{16}{3}+\frac{7}{3}=\boxed{\frac{23}{3}}$$

27) $\int_0^{\pi}(1+\sin x)\,dx=\left.x-\cos x\right]_0^{\pi}=\pi-\cos\pi-(0-\cos 0)$

$$=\pi-(-1)+1=\boxed{\pi+2}$$

29) $\int_{-\pi/6}^{\pi/6}\sec^2 x\,dx=\left.\tan x\right]_{-\pi/6}^{\pi/6}=\tan\pi/6-\tan(-\pi/6)=\frac{1}{\sqrt{3}}-\left(-\frac{1}{\sqrt{3}}\right)=\frac{2}{\sqrt{3}}=\boxed{\frac{2\sqrt{3}}{3}}$

31) $\int_{-\pi/3}^{\pi/3}4\sec\theta\tan\theta\,d\theta=\left.4\sec\theta\right]_{-\pi/3}^{\pi/3}=4\sec(\pi/3)-4\sec(-\pi/3)$

$$=4(2)-4(2)$$

$$=8-8=\boxed{0}$$