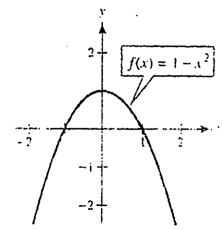
75. The function $f$ is nonnegative between $x = -1$ and $x = 1$.

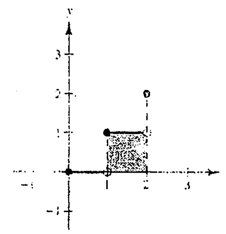
So, $\int_a^b (1 - x^2)\,dx$

is a maximum for

$a = -1$ and $b = 1$.




76. To find $\int_0^2 [\![x]\!]\,dx$, use a geometric approach.



So, $\int_0^2 [\![x]\!]\,dx = 1(2 - 1) = 1$.

77. Let $f(x) = x^2$, $0 \le x \le 1$, and $\Delta x_i = 1/n$. The appropriate Riemann Sum is

$$\sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\frac{1}{n} = \frac{1}{n^3}\sum_{i=1}^{n} i^2.$$

$$\lim_{n\to\infty}\frac{1}{n^3}\left[1^2 + 2^2 + 3^2 + \cdots + n^2\right] = \lim_{n\to\infty}\frac{1}{n^3}\cdot\frac{n(2n+1)(n+1)}{6} = \lim_{n\to\infty}\frac{2n^2 + 3n + 1}{6n^2} = \lim_{n\to\infty}\left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right) = \frac{1}{3}$$

78. $I(f) - J(f) = \int_0^1 x^2 f(x)\,dx - \int_0^1 xf(x)^2\,dx.$

Observe that

$$\frac{x^3}{4} - x\left(f(x) - \frac{x}{2}\right)^2 = \frac{x^3}{4} - x\left(f(x)^2 - xf(x) + \frac{x^2}{4}\right) = \frac{x^3}{4} - xf(x)^2 + x^2f(x) - \frac{x^3}{4} = x^2f(x) - xf(x)^2$$

$$\text{So, } I(f) - J(f) = \int_0^1\left[x^2f(x) - xf(x)^2\right]dx = \int_0^1\left[\frac{x^3}{4} - x\left(f(x) - \frac{x}{2}\right)^2\right]dx \le \int_0^1\frac{x^3}{4}\,dx = \frac{1}{16}$$

Furthermore, $6 + f(x) = \frac{x}{2}$. Then $I(f) = \int_0^1 x^2\left(\frac{x}{2}\right)dx = \frac{1}{8}$ and $J(f) = \int_0^1 x\left(\frac{x^2}{4}\right) = \frac{1}{16}$

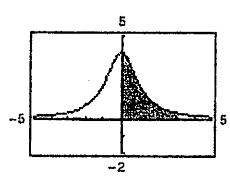
So $I(f) - J(f) = \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$

The maximum value is $\frac{1}{16}$.

## Section 4.4 The Fundamental Theorem of Calculus

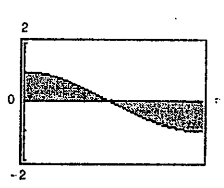
1. $f(x) = \dfrac{4}{x^2 + 1}$

$\int_0^\pi \dfrac{4}{x^2 + 1}\,dx$ is positive.



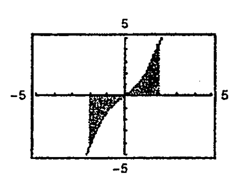
2. $f(x) = \cos x$

$\int_0^\pi \cos x\,dx = 0$



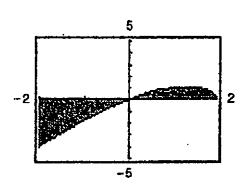
3. $f(x) = x\sqrt{x^2 + 1}$

$\int_{-2}^2 x\sqrt{x^2 + 1}\,dx = 0$



4. $f(x) = x\sqrt{2 - x}$

$\int_{-2}^2 x\sqrt{2 - x}\,dx$ is negative.



5. $\int_0^2 6x\,dx = \left[3x^2\right]_0^2 = 3(2)^2 - 0$

6. $\int_{-3}^1 8\,dt = \left[8t\right]_{-3}^1 = 8(1) - 8(-3) = 32$

7. $\int_{-1}^0 (2x - 1)\,dx = \left[x^2 - x\right]_{-1}^0$

$= 0 - \left((-1)^2 - (-1)\right) = -(1 + 1) = -2$

8. $\int_{-1}^2 (7 - 3t)\,dt = \left[7t - \frac{3}{2}t^2\right]_{-1}^2$

$= \left[7(2) - \frac{3}{2}(4)\right] - \left[7(-1) - \frac{3}{2}(-1)^2\right]$

$= 14 - 6 + 7 + \frac{3}{2} = \frac{33}{2}$

9. $\int_{-1}^1 (t^2 - 2)\,dt = \left[\frac{t^3}{3} - 2t\right]_{-1}^1$

$= \left(\frac{1}{3} - 2\right) - \left(-\frac{1}{3} + 2\right) = -\frac{10}{3}$

10. $\int_1^2 (6x^2 - 3x)\,dx = \left[2x^3 - \frac{3}{2}x^2\right]_1^2 = \left[2(8) - \frac{3}{2}(4)\right] - \left[2(1) - \frac{3}{2}(1)\right] = (16 - 6) - \left(2 - \frac{3}{2}\right) = \frac{19}{2}$

11. $\int_0^1 (2t - 1)^2\,dt = \int_0^1 (4t^2 - 4t + 1)\,dt = \left[\frac{4}{3}t^3 - 2t^2 + t\right]_0^1 = \frac{4}{3} - 2 + 1 = \frac{1}{3}$

12. $\int_1^3 (4x^3 - 3x^2)\,dx = \left[x^4 - x^3\right]_1^3 = (81 - 27) - (1 - 1) = 54$

13. $\int_1^2 \left(\frac{3}{x^2} - 1\right)dx = \left[-\frac{3}{x} - x\right]_1^2 = \left(-\frac{3}{2} - 2\right) - (-3 - 1) = \frac{1}{2}$

14. $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right)du = \left[\frac{u^2}{2} + \frac{1}{u}\right]_{-2}^{-1} = \left(\frac{1}{2} - 1\right) - \left(2 - \frac{1}{2}\right) = -2$

15. $\int_1^4 \frac{u - 2}{\sqrt{u}}\,du = \int_1^4 \left(u^{1/2} - 2u^{-1/2}\right)du = \left[\frac{2}{3}u^{3/2} - 4u^{1/2}\right]_1^4 = \left[\frac{2}{3}\left(\sqrt{4}\right)^3 - 4\sqrt{4}\right] - \left[\frac{2}{3} - 4\right] = \frac{2}{3}$

16. $\int_{-8}^8 x^{1/3}\,dx = \left[\frac{3}{4}x^{4/3}\right]_{-8}^8 = \frac{3}{4}\left[8^{4/3} - (-8)^{4/3}\right] = \frac{3}{4}(16 - 16) = 0$

17. $\int_{-1}^1 \left(\sqrt[3]{t} - 2\right)dt = \left[\frac{3}{4}t^{4/3} - 2t\right]_{-1}^1 = \left(\frac{3}{4} - 2\right) - \left(\frac{3}{4} + 2\right) = -4$

18. $\int_1^8 \sqrt{\frac{2}{x}}\,dx = \sqrt{2}\int_1^8 x^{-1/2}\,dx = \left[\sqrt{2}(2)x^{1/2}\right]_1^8 = \left[2\sqrt{2x}\right]_1^8 = 8 - 2\sqrt{2}$

19. $\int_0^1 \frac{x - \sqrt{x}}{3}\,dx = \frac{1}{3}\int_0^1 \left(x - x^{1/2}\right)dx = \frac{1}{3}\left[\frac{x^2}{2} - \frac{2}{3}x^{3/2}\right]_0^1 = \frac{1}{3}\left(\frac{1}{2} - \frac{2}{3}\right) = -\frac{1}{18}$

20. $\int_0^2 (2 - t)\sqrt{t}\,dt = \int_0^2 \left(2t^{1/2} - t^{3/2}\right)dt = \left[\frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^2 = \left[\frac{t\sqrt{t}}{15}(20 - 6t)\right]_0^2 = \frac{2\sqrt{2}}{15}(20 - 12) = \frac{16\sqrt{2}}{15}$

21. $\int_{-1}^0 \left(t^{1/3} - t^{2/3}\right)dt = \left[\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right]_{-1}^0 = 0 - \left(\frac{3}{4} + \frac{3}{5}\right) = -\frac{27}{20}$

22. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx = \frac{1}{2}\int_{-8}^{-1} \left(x^{2/3} - x^{5/3}\right)dx$

$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1} = \left[\frac{x^{5/3}}{80}(24 - 15x)\right]_{-8}^{-1} = -\frac{1}{80}(39) + \frac{32}{80}(144) = \frac{4569}{80}$

**23.** $\int_0^5 |2x - 5|\,dx = \int_0^{5/2} (5 - 2x)\,dx + \int_{5/2}^5 (2x - 5)\,dx$ (split up the integral at the zero $x = \frac{5}{2}$)

$= \left[5x - x^2\right]_0^{5/2} + \left[x^2 - 5x\right]_{5/2}^5 = \left(\frac{25}{2} - \frac{25}{4}\right) - 0 + (25 - 25) - \left(\frac{25}{4} - \frac{25}{2}\right) = 2\left(\frac{25}{2} - \frac{25}{4}\right) = \frac{25}{2}$

**Note:** By Symmetry, $\int_0^5 |2x - 5|\,dx = 2\int_{5/2}^5 (2x - 5)\,dx.$

**24.** $\int_1^4 (3 - |x - 3|)\,dx = \int_1^3 \left[3 + (x - 3)\right]dx + \int_3^4 \left[3 - (x - 3)\right]dx$

$= \int_1^3 x\,dx + \int_3^4 (6 - x)dx$

$= \left[\frac{x^2}{2}\right]_1^3 + \left[6x - \frac{x^2}{2}\right]_3^4$

$= \left(\frac{9}{2} - \frac{1}{2}\right) + \left[(24 - 8) - \left(18 - \frac{9}{2}\right)\right]$

$= 4 + 16 - 18 + \frac{9}{2} = \frac{13}{2}$

**25.** $\int_0^4 \left|x^2 - 9\right|dx = \int_0^3 \left(9 - x^2\right)dx + \int_3^4 \left(x^2 - 9\right)dx$ (split up integral at the zero $x = 3$)

$= \left[9x - \frac{x^3}{3}\right]_0^3 + \left[\frac{x^3}{3} - 9x\right]_3^4 = (27 - 9) + \left(\frac{64}{3} - 36\right) - (9 - 27) = \frac{64}{3}$

**26.** $\int_0^4 \left|x^2 - 4x + 3\right|dx = \int_0^1 \left(x^2 - 4x + 3\right)dx - \int_1^3 \left(x^2 - 4x + 3\right)dx + \int_3^4 \left(x^2 - 4x + 3\right)dx$ (split up the integral at the zeros $x = 1, 3$)

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3 + \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^4$

$= \left(\frac{1}{3} - 2 + 3\right) - (9 - 18 + 9) + \left(\frac{1}{3} - 2 + 3\right) + \left(\frac{64}{3} - 32 + 12\right) - (9 - 18 + 9)$

$= \frac{4}{3} - 0 + \frac{4}{3} + \frac{4}{3} - 0 = 4$

**27.** $\int_0^\pi (1 + \sin x)\,dx = \left[x - \cos x\right]_0^\pi = (\pi + 1) - (0 - 1) = 2 + \pi$

**28.** $\int_0^\pi (2 + \cos x)\,dx = \left[2x + \sin x\right]_0^\pi = (2\pi + 0) - 0 = 2\pi$

**29.** $\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} d\theta = \left[\theta\right]_0^{\pi/4} = \frac{\pi}{4}$

**30.** $\int_0^{\pi/4} \frac{\sec^2\theta}{\tan^2\theta + 1}\,d\theta = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/4} d\theta = \left[\theta\right]_0^{\pi/4} = \frac{\pi}{4}$

**31.** $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx = \left[\tan x\right]_{-\pi/6}^{\pi/6} = \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}$

**32.** $\int_{\pi/4}^{\pi/2} \left(2 - \csc^2 x\right)dx = \left[2x + \cot x\right]_{\pi/4}^{\pi/2} = (\pi + 0) - \left(\frac{\pi}{2} + 1\right) = \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

**33.** $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta = \left[4\sec\theta\right]_{-\pi/3}^{\pi/3} = 4(2) - 4(2) = 0$

**34.** $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\, dt = \left[t^2 + \sin t\right]_{-\pi/2}^{\pi/2} = \left(\dfrac{\pi^2}{4} + 1\right) - \left(\dfrac{\pi^2}{4} - 1\right) = 2$

**35.** $A = \int_0^1 \left(x - x^2\right) dx = \left[\dfrac{x^2}{2} - \dfrac{x^3}{3}\right]_0^1 = \dfrac{1}{6}$

**36.** $A = \int_1^2 \dfrac{1}{x^2}\, dx = \left[-\dfrac{1}{x}\right]_1^2 = \dfrac{1}{2}$

**37.** $A = \int_0^{\pi/2} \cos x\, dx = \left[\sin x\right]_0^{\pi/2} = 1$

**38.** $A = \int_0^{\pi} (x + \sin x)\, dx = \left[\dfrac{x^2}{2} - \cos x\right]_0^{\pi} = \dfrac{\pi^2}{2} + 2 = \dfrac{\pi^2 + 4}{2}$

**39.** Because $y > 0$ on $[0, 2]$,

Area $= \int_0^2 \left(5x^2 + 2\right) dx = \left[\frac{5}{3}x^3 + 2x\right]_0^2 = \frac{40}{3} + 4 = \frac{52}{3}.$

**40.** Because $y > 0$ on $[0, 2]$,

Area $= \int_0^2 \left(x^3 + x\right) dx = \left[\dfrac{x^4}{4} + \dfrac{x^2}{2}\right]_0^2 = 4 + 2 = 6.$

**41.** Because $y > 0$ on $[0, 8]$,

Area $= \int_0^8 \left(1 + x^{1/3}\right) dx = \left[x + \dfrac{3}{4}x^{4/3}\right]_0^8 = 8 + \dfrac{3}{4}(16) = 20.$

**42.** Because $y \ge 0$ on $[0, 4]$,

$$\text{Area} = \int_0^4 \left(2\sqrt{x} - x\right) dx$$

$$= \int_0^4 \left(2x^{1/2} - x\right) dx = \left[\dfrac{4}{3}x^{3/2} - \dfrac{x^2}{2}\right]_0^4 = \dfrac{32}{3} - 8 = \dfrac{8}{3}.$$

**43.** Because $y > 0$ on $[0, 4]$,

Area $= \int_0^4 \left(-x^2 + 4x\right) dx = \left[-\dfrac{x^3}{3} + 2x^2\right]_0^4 = -\dfrac{64}{3} + 32 = \dfrac{32}{3}.$

**44.** Because $y > 0$ on $[-1, 1]$,

$$\text{Area} = \int_{-1}^1 \left(1 - x^4\right) dx$$

$$= 2\int_0^1 \left(1 - x^4\right) dx$$

$$= 2\left[x - \dfrac{x^5}{5}\right]_0^1 = 2\left(1 - \dfrac{1}{5}\right) = \dfrac{8}{5}.$$

**45.** $\int_0^3 x^3\,dx = \left[\frac{x^4}{4}\right]_0^3 = \frac{81}{4}$

$$f(c)(3-0) = \frac{81}{4}$$
$$f(c) = \frac{27}{4}$$
$$c^3 = \frac{27}{4}$$
$$c = \frac{3}{\sqrt[3]{4}} = \frac{3}{2}\sqrt[3]{2} \approx 1.8899$$

**46.** $\int_4^9 \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_4^9 = \frac{2}{3}(27-8) = \frac{38}{3}$

$$f(c)(9-4) = \frac{38}{3}$$
$$f(c) = \frac{38}{15}$$
$$\sqrt{c} = \frac{38}{15}$$
$$c = \frac{1444}{225} \approx 6.4178$$

**47.** $\int_0^6 \frac{x^2}{4}\,dx = \left[\frac{x^3}{12}\right]_0^6 = \frac{216}{12} = 18$

$$f(c)(6-0) = 18$$
$$f(c) = 3$$
$$\frac{c^2}{4} = 3$$
$$c = \pm\sqrt{12}$$
$$c = 2\sqrt{3}$$

$\left(-2\sqrt{3}\text{ is not in interval.}\right)$

**48.** $\int_1^3 \frac{9}{x^3}\,dx = \left[-\frac{9}{2x^2}\right]_1^3 = -\frac{1}{2} + \frac{9}{2} = 4$

$$f(c)(3-1) = 4$$
$$\frac{9}{c^3} = 2$$
$$c^3 = \frac{9}{2}$$
$$c = \sqrt[3]{\frac{9}{2}} \approx 1.6510$$

**49.** $\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = \left[2\tan x\right]_{-\pi/4}^{\pi/4} = 2(1) - 2(-1) = 4$

$$f(c)\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = 4$$
$$2\sec^2 c = \frac{8}{\pi}$$
$$\sec^2 c = \frac{4}{\pi}$$
$$\sec c = \pm\frac{2}{\sqrt{\pi}}$$
$$c = \pm\operatorname{arcsec}\left(\frac{2}{\sqrt{\pi}}\right)$$
$$= \pm\arccos\frac{\sqrt{\pi}}{2} \approx \pm 0.4817$$

**50.** $\int_{-\pi/3}^{\pi/3} \cos x\,dx = \left[\sin x\right]_{-\pi/3}^{\pi/3} = \sqrt{3}$

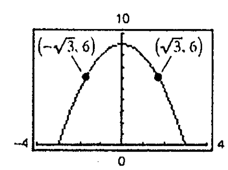
$$f(c)\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \sqrt{3}$$
$$\cos c = \frac{3\sqrt{3}}{2\pi}$$
$$c \approx \pm 0.5971$$

**51.** $\frac{1}{3-(-3)}\int_{-3}^{3}\left(9-x^2\right)dx = \frac{1}{6}\left[9x - \frac{1}{3}x^3\right]_{-3}^{3}$

$$= \frac{1}{6}\left[(27-9) - (-27+9)\right]$$
$$= 6$$

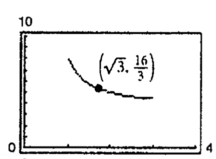
Average value = 6

$9 - x^2 = 6$ when $x^2 = 9 - 6$ or $x = \pm\sqrt{3} \approx \pm 1.7321$.




**52.** $\frac{1}{3-1}\int_1^3 \frac{4\left(x^2+1\right)}{x^2}\,dx = 2\int_1^3\left(1 + x^{-2}\right)dx$

$$= 2\left[x - \frac{1}{x}\right]_1^3$$
$$= 2\left(3 - \frac{1}{3}\right) = \frac{16}{3}$$




Average value $= \frac{16}{3}$

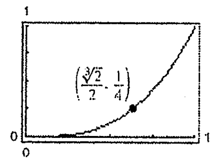
$\frac{4\left(x^2+1\right)}{x^2} = \frac{16}{3} \Rightarrow x = \sqrt{3}$ (on $[1, 3]$)

**53.** $\dfrac{1}{1-0}\int_0^1 x^3\,dx = \left[\dfrac{x^4}{4}\right]_0^1 = \dfrac{1}{4}$

Average value $= \dfrac{1}{4}$

$x^3 = \dfrac{1}{4}$

$x = \sqrt[3]{\dfrac{1}{4}} = \dfrac{\sqrt[3]{2}}{2} \approx 0.6300$



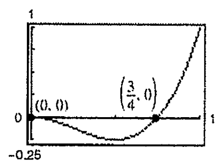
**54.** $\dfrac{1}{1-0}\int_0^1 \left(4x^3 - 3x^2\right) dx = \left[x^4 - x^3\right]_0^1 = 0$

Average value $= 0$

$4x^3 - 3x^2 = 0$

$x^2(4x - 3) = 0$

$x = 0, \frac{3}{4}$

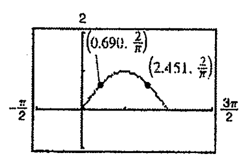


**55.** $\dfrac{1}{\pi - 0}\int_0^{\pi} \sin x\,dx = \left[-\dfrac{1}{\pi}\cos x\right]_0^{\pi} = \dfrac{2}{\pi}$

Average value $= \dfrac{2}{\pi}$

$\sin x = \dfrac{2}{\pi}$

$x \approx 0.690, 2.451$

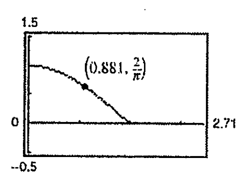


**56.** $\dfrac{1}{(\pi/2) - 0}\int_0^{\pi/2} \cos x\,dx = \left[\dfrac{2}{\pi}\sin x\right]_0^{\pi/2} = \dfrac{2}{\pi}$

Average value $= \dfrac{2}{\pi}$

$\cos x = \dfrac{2}{\pi}$

$x \approx 0.881$



**57.** The distance traveled is $\int_0^8 v(t)\,dt$. The area under the curve from $0 \le t \le 8$ is approximately (18 squares) $(30) \approx 540$ ft.

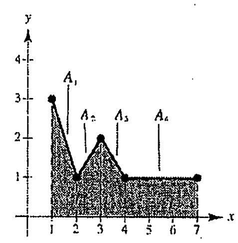
**58.** The distance traveled is $\int_0^5 v(t)\,dt$. The area under the curve from $0 \le t \le 5$ is approximately (29 squares) $(5) = 145$ ft.

**59.** (a) $\int_1^7 f(x)\,dx =$ Sum of the areas

$= A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2}(3+1) + \frac{1}{2}(1+2) + \frac{1}{2}(2+1) + (3)(1)$

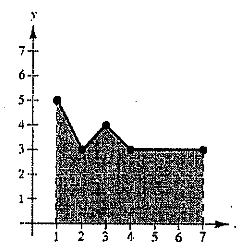
$= 8$



(b) Average value $= \dfrac{\int_1^7 f(x)\,dx}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}$

(c) $A = 8 + (6)(2) = 20$

Average value $= \dfrac{20}{6} = \dfrac{10}{3}$



**60.** $r(t)$ represents the weight in pounds of the dog at time $t$.

$\int_2^6 r'(t)\,dt$ represents the net change in the weight of the dog from year 2 to year 6.

**61.** (a) $F(x) = k\sec^2 x$

$F(0) = k = 500$

$F(x) = 500\sec^2 x$

(b) $\dfrac{1}{\pi/3 - 0}\int_0^{\pi/3} 500\sec^2 x\,dx = \dfrac{1500}{\pi}\left[\tan x\right]_0^{\pi/3}$

$= \dfrac{1500}{\pi}\left(\sqrt{3} - 0\right)$

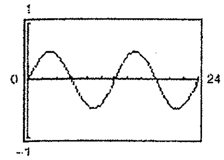
$\approx 826.99$ newtons

$\approx 827$ newtons

**62.** $\dfrac{1}{R-0}\int_0^R k\left(R^2 - r^2\right) dr = \dfrac{k}{R}\left[R^2 r - \dfrac{r^3}{3}\right]_0^R = \dfrac{2kR^2}{3}$

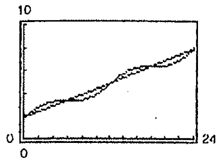
**63.** $\frac{1}{5-0}\int_0^5 \left(0.1729t + 0.1522t^2 - 0.0374t^3\right) dt \approx \frac{1}{5}\Big[0.08645t^2 + 0.05073t^3 - 0.00935t^4\Big]_0^5 \approx 0.5318$ liter

**64.** (a)




The area above the $x$-axis equals the area below the $x$-axis. So, the average value is zero.

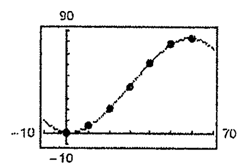
(b)




The average value of $S$ appears to be $g$.

**65.** (a) $v = -0.00086t^3 + 0.0782t^2 - 0.208t + 0.10$

(b)




(c) $\int_0^{60} v(t)\,dt = \left[\frac{-0.00086t^4}{4} + \frac{0.0782t^3}{3} - \frac{0.208t^2}{2} + 0.10t\right]_0^{60} \approx 2476$ meters

**66.** (a) Because $y < 0$ on $[0, 2]$, $\int_0^2 f(x)\,dx = -(\text{area of region } A) = -1.5$.

(b) $\int_2^6 f(x)\,dx = (\text{area of region } B) = \int_0^6 f(x)\,dx - \int_0^2 f(x)\,dx = 3.5 - (-1.5) = 5.0$

(c) $\int_0^6 |f(x)|\,dx = -\int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 1.5 + 5.0 = 6.5$

(d) $\int_0^2 -2f(x)\,dx = -2\int_0^2 f(x)\,dx = -2(-1.5) = 3.0$

(e) $\int_0^6 \left[2 + f(x)\right]dx = \int_0^6 2\,dx + \int_0^6 f(x)\,dx = 12 + 3.5 = 15.5$

(f) Average value $= \frac{1}{6}\int_0^6 f(x)\,dx = \frac{1}{6}(3.5) = 0.5833$

**67.** $F(x) = \int_0^x (4t - 7)\,dt = \left[2t^2 - 7t\right]_0^x = 2x^2 - 7x$

$F(2) = 2(2^2) - 7(2) = -6$

$F(5) = 2(5^2) - 7(5) = 15$

$F(8) = 2(8^2) - 7(8) = 72$

**68.** $F(x) = \int_2^x (t^3 + 2t - 2)\,dt = \left[\frac{t^4}{4} + t^2 - 2t\right]_2^x = \left(\frac{x^4}{4} + x^2 - 2x\right) - (4 + 4 - 4) = \frac{x^4}{4} + x^2 - 2x - 4$

$F(2) = 4 + 4 - 4 - 4 = 0$ $\left[\textbf{Note: } F(2) = \int_2^2 (t^3 + 2t - 2)\,dt = 0\right]$

$F(5) = \frac{625}{4} + 25 - 10 - 4 = 167.25$

$F(8) = \frac{8^4}{4} + 64 - 16 - 4 = 1068$

**69.** $F(x) = \int_1^x \frac{20}{v^2}\,dv = \int_1^x 20v^{-2}\,dv = -\frac{20}{v}\Big]_1^x$

$= -\frac{20}{x} + 20 = 20\left(1 - \frac{1}{x}\right)$

$F(2) = 20\left(\frac{1}{2}\right) = 10$

$F(5) = 20\left(\frac{4}{5}\right) = 16$

$F(8) = 20\left(\frac{7}{8}\right) = \frac{35}{2}$

**70.** $F(x) = \int_2^x -\frac{2}{t^3}\,dt = -\int_2^x 2t^{-3}\,dt = \frac{1}{t^2}\Big]_2^x = \frac{1}{x^2} - \frac{1}{4}$

$F(2) = \frac{1}{4} - \frac{1}{4} = 0$

$F(5) = \frac{1}{25} - \frac{1}{4} = -\frac{21}{100} = -0.21$

$F(8) = \frac{1}{64} - \frac{1}{4} = -\frac{15}{64}$

**71.** $F(x) = \int_1^x \cos\theta\,d\theta = \sin\theta\Big]_1^x = \sin x - \sin 1$

$F(2) = \sin 2 - \sin 1 \approx 0.0678$

$F(5) = \sin 5 - \sin 1 \approx -1.8004$

$F(8) = \sin 8 - \sin 1 \approx 0.1479$

**72.** $F(x) = \int_0^x \sin\theta\,d\theta = -\cos\theta\Big]_0^x$

$= -\cos x + \cos 0$

$= 1 - \cos x$

$F(2) = 1 - \cos 2 \approx 1.4161$

$F(5) = 1 - \cos 5 \approx 0.7163$

$F(8) = 1 - \cos 8 \approx 1.1455$

**73.** $g(x) = \int_0^x f(t)\,dt$

(a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(2) = \int_0^2 f(t)\,dt \approx 4 + 2 + 1 = 7$

$g(4) = \int_0^4 f(t)\,dt \approx 7 + 2 = 9$

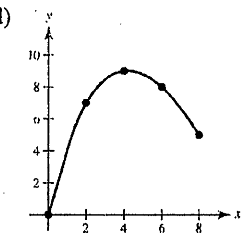
$g(6) = \int_0^6 f(t)\,dt \approx 9 + (-1) = 8$

$g(8) = \int_0^8 f(t)\,dt \approx 8 - 3 = 5$

(b) $g$ increasing on $(0, 4)$ and decreasing on $(4, 8)$

(c) $g$ is a maximum of 9 at $x = 4$.

(d)




**74.** $g(x) = \int_0^x f(t)\,dt$

(a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(2) = \int_0^2 f(t)\,dt = -\frac{1}{2}(2)(4) = -4$

$g(4) = \int_0^4 f(t)\,dt = -\frac{1}{2}(4)(4) = -8$

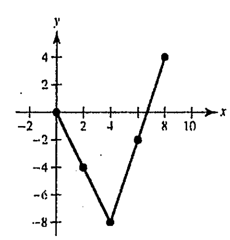
$g(6) = \int_0^6 f(t)\,dt = -8 + 2 + 4 = -2$

$g(8) = \int_0^8 f(t)\,dt = -2 + 6 = 4$

(b) $g$ decreasing on $(0, 4)$ and increasing on $(4, 8)$

(c) $g$ is a minimum of $-8$ at $x = 4$.

(d)



**75.** (a) $\int_0^x (t+2)\,dt = \left[\frac{t^2}{2} + 2t\right]_0^x = \frac{1}{2}x^2 + 2x$

(b) $\frac{d}{dx}\left[\frac{1}{2}x^2 + 2x\right] = x + 2$

**76.** (a) $\int_0^x t(t^2+1)\,dt = \int_0^x (t^3 + t)\,dt$

$= \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^x$

$= \frac{1}{4}x^4 + \frac{1}{2}x^2 = \frac{x^2}{4}(x^2 + 2)$

(b) $\frac{d}{dx}\left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right] = x^3 + x = x(x^2+1)$

**77.** (a) $\int_8^x \sqrt[3]{t}\,dt = \left[\frac{3}{4}t^{4/3}\right]_8^x = \frac{3}{4}(x^{4/3} - 16) = \frac{3}{4}x^{4/3} - 12$

(b) $\frac{d}{dx}\left[\frac{3}{4}x^{4/3} - 12\right] = x^{1/3} = \sqrt[3]{x}$

**78.** (a) $\int_4^x \sqrt{t}\,dt = \left[\frac{2}{3}t^{3/2}\right]_4^x$

$= \frac{2}{3}x^{3/2} - \frac{16}{3}$

$= \frac{2}{3}(x^{3/2} - 8)$

(b) $\frac{d}{dx}\left[\frac{2}{3}x^{3/2} - \frac{16}{3}\right] = x^{1/2} = \sqrt{x}$

**79.** (a) $\int_{\pi/4}^x \sec^2 t\,dt = [\tan t]_{\pi/4}^x = \tan x - 1$

(b) $\frac{d}{dx}[\tan x - 1] = \sec^2 x$

**80.** (a) $\int_{\pi/3}^x \sec t \tan t\,dt = [\sec t]_{\pi/3}^x = \sec x - 2$

(b) $\frac{d}{dx}[\sec x - 2] = \sec x \tan x$

**81.** $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

$F'(x) = x^2 - 2x$

**82.** $F(x) = \int_1^x \frac{t^2}{t^2+1}\,dt$

$F'(x) = \frac{x^2}{x^2+1}$

**83.** $F(x) = \int_{-1}^x \sqrt{t^4+1}\,dt$

$F'(x) = \sqrt{x^4+1}$

**84.** $F(x) = \int_1^x \sqrt[4]{t}\,dt$

$F'(x) = \sqrt[4]{x}$

**85.** $F(x) = \int_0^x t\cos t\,dt$

$F'(x) = x\cos x$

**86.** $F(x) = \int_0^x \sec^3 t\,dt$

$F'(x) = \sec^3 x$

**87.** $F(x) = \int_x^{x+2} (4t+1)\,dt$

$= \left[2t^2 + t\right]_x^{x+2}$

$= \left[2(x+2)^2 + (x+2)\right] - \left[2x^2 + x\right]$

$= 8x + 10$

$F'(x) = 8$

**Alternate solution:**

$F(x) = \int_x^{x+2} (4t+1)\,dt$

$= \int_x^0 (4t+1)\,dt + \int_0^{x+2} (4t+1)\,dt$

$= -\int_0^x (4t+1)\,dt + \int_0^{x+2} (4t+1)\,dt$

$F'(x) = -(4x+1) + 4(x+2) + 1 = 8$

**88.** $F(x) = \int_{-x}^x t^3\,dt = \left[\frac{t^4}{4}\right]_{-x}^x = 0$

$F'(x) = 0$

**Alternate solution:**

$F(x) = \int_{-x}^x t^3\,dt$

$= \int_{-x}^0 t^3\,dt + \int_0^x t^3\,dt$

$= -\int_0^{-x} t^3\,dt + \int_0^x t^3\,dt$

$F'(x) = -(-x)^3(-1) + (x^3) = 0$

**89.** $F(x) = \int_0^{\sin x} \sqrt{t}\,dt = \left[\frac{2}{3}t^{3/2}\right]_0^{\sin x} = \frac{2}{3}(\sin x)^{3/2}$

$F'(x) = (\sin x)^{1/2}\cos x = \cos x\sqrt{\sin x}$

**Alternate solution:**

$F(x) = \int_0^{\sin x} \sqrt{t}\,dt$

$F'(x) = \sqrt{\sin x}\frac{d}{dx}(\sin x) = \sqrt{\sin x}(\cos x)$

**90.** $F(x) = \int_2^{x^2} t^{-3}\,dt = \left[\frac{t^{-2}}{-2}\right]_2^{x^2} = \left[-\frac{1}{2t^2}\right]_2^{x^2} = \frac{-1}{2x^4} + \frac{1}{8}$

$F'(x) = 2x^{-5}$

**Alternate solution:**

$F(x) = \int_2^{x^2} t^{-3}\,dt$

$F'(x) = (x^2)^{-3}(2x) = 2x^{-5}$

**91.** $F(x) = \int_0^{x^3} \sin t^2\,dt$

$F'(x) = \sin(x^3)^2 \cdot 3x^2 = 3x^2 \sin x^6$

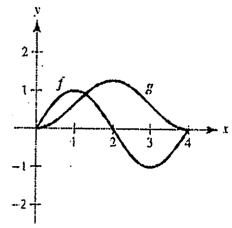
**92.** $F(x) = \int_0^{x^2} \sin \theta^2\,d\theta$

$F'(x) = \sin(x^2)^2(2x) = 2x \sin x^4$

**93.** $g(x) = \int_0^x f(t)\,dt$

$g(0) = 0, g(1) \approx \frac{1}{2}, g(2) \approx 1, g(3) \approx \frac{1}{2}, g(4) = 0$

$g$ has a relative maximum at $x = 2$.




**94.** (a) $g(t) = 4 - \frac{4}{t^2}$

$\lim_{t\to\infty} g(t) = 4$

Horizontal asymptote: $y = 4$

(b) $A(x) = \int_1^x \left(4 - \frac{4}{t^2}\right)dt = \left[4t + \frac{4}{t}\right]_1^x = 4x + \frac{4}{x} - 8 = \frac{4x^2 - 8x + 4}{x} = \frac{4(x-1)^2}{x}$

$\lim_{x\to\infty} A(x) = \lim_{x\to\infty}\left(4x + \frac{4}{x} - 8\right) = \infty + 0 - 8 = \infty$

The graph of $A(x)$ does not have a horizontal asymptote.

**95.** (a) $v(t) = 5t - 7,\ 0 \le t \le 3$

$\text{Displacement} = \int_0^3 (5t - 7)\,dt = \left[\frac{5t^2}{2} - 7t\right]_0^3 = \frac{45}{2} - 21 = \frac{3}{2}$ ft to the right

(b) $\text{Total distance traveled} = \int_0^3 |5t - 7|\,dt$

$= \int_0^{7/5} (7 - 5t)\,dt + \int_{7/5}^3 (5t - 7)\,dt$

$= \left[7t - \frac{5t^2}{2}\right]_0^{7/5} + \left[\frac{5t^2}{2} - 7t\right]_{7/5}^3$

$= 7\left(\frac{7}{5}\right) - \frac{5}{2}\left(\frac{7}{2}\right)^2 + \left(\frac{5}{2}(9) - 21\right) - \left(\frac{5}{2}\left(\frac{7}{5}\right)^2 - 7\left(\frac{7}{5}\right)\right)$

$= \frac{49}{5} - \frac{49}{10} + \frac{45}{2} - 21 - \frac{49}{10} + \frac{49}{5} = \frac{113}{10}$ ft

**96.** (a) $v(t) = t^2 - t - 12 = (t - 4)(t + 3),\ 1 \le t \le 5$

$\text{Displacement} = \int_1^5 (t^2 - t - 12)\,dt$

$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 12t\right]_1^5 = \left(\frac{125}{3} - \frac{25}{2} - 60\right) - \left(\frac{1}{3} - \frac{1}{2} - 12\right) = -\frac{56}{3}$ $\left(\frac{56}{3}\text{ ft to the left}\right)$

(b) Total distance traveled $= \int_1^4 \left(-t^2 + t + 12\right) dt + \int_4^5 \left(t^2 - t - 12\right) dt$

$$= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 12t\right]_1^4 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 12t\right]_4^5$$

$$= \left(-\frac{64}{3} + 8 + 48\right) - \left(-\frac{1}{3} + \frac{1}{2} + 12\right) + \left(\frac{125}{3} - \frac{25}{2} - 60\right) - \left(\frac{64}{3} - 8 - 48\right)$$

$$= \frac{104}{3} - \frac{73}{6} + \left(-\frac{185}{6}\right) - \left(-\frac{104}{3}\right) = \frac{79}{3} \text{ ft}$$

**97.** (a) $v(t) = t^3 - 10t^2 + 27t - 18 = (t - 1)(t - 3)(t - 6),\ 1 \le t \le 7$

Displacement $= \int_1^7 \left(t^3 - 10t^2 + 27t - 18\right) dt$

$$= \left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{27t^2}{2} - 18t\right]_1^7$$

$$= \left[\frac{7^4}{4} - \frac{10(7^3)}{3} + \frac{27(7^2)}{2} - 18(7)\right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{27}{2} - 18\right]$$

$$= -\frac{91}{12} - \left(-\frac{91}{12}\right) = 0$$

(b) Total distance traveled $= \int_1^7 \left|v(t)\right| dt$

$$= \int_1^3 \left(t^3 - 10t^2 + 27t - 18\right) dt - \int_3^6 \left(t^3 - 10t^2 + 27t - 18\right) dt + \int_6^7 \left(t^3 - 10t^2 + 27t - 18\right) dt$$

Evaluating each of these integrals, you obtain

Total distance $= \frac{16}{3} - \left(-\frac{63}{4}\right) + \frac{125}{12} = \frac{63}{2}$ ft

**98.** (a) $v(t) = t^3 - 8t^2 + 15t = t(t - 3)(t - 5),\ 0 \le t \le 5$

Displacement $= \int_0^5 \left(t^3 - 8t^2 + 15t\right) dt$

$$= \left[\frac{t^4}{4} - \frac{8t^3}{3} + \frac{15t^2}{2}\right]_0^5$$

$$= \frac{625}{4} - \frac{8(125)}{3} + \frac{375}{2} = \frac{125}{12} \text{ ft to the right}$$

(b) Total distance traveled $= \int_0^5 \left|v(t)\right| dt$

$$= \int_0^3 \left(t^3 - 8t^2 + 15t\right) dt - \int_3^5 \left(t^3 - 8t^2 + 15t\right) dt$$

Evaluating each of these integrals, you obtain

Total distance $= \frac{63}{4} - \left(-\frac{16}{3}\right) = \frac{253}{12} \approx 21.08$ ft

**99.** (a) $v(t) = \frac{1}{\sqrt{t}},\ 1 \le t \le 4$

Because $v(t) > 0$,

Displacement = Total Distance

Displacement $= \int_1^4 t^{-1/2}\, dt = \left[2t^{1/2}\right]_1^4 = 4 - 2 = 2$ ft to the right

(b) Total distance = 2 ft

**100.** (a) $v(t) = \cos t,\ 0 \le t \le 3\pi$

Displacement $= \int_0^{3\pi} \cos t\, dt = \left[\sin t\right]_0^{3\pi} = 0$ ft

(b) Total distance $= \int_0^{\pi/2} \cos t\, dt - \int_{\pi/2}^{3\pi/2} \cos t\, dt + \int_{3\pi/2}^{5\pi/2} \cos t\, dt - \int_{5\pi/2}^{3\pi} \cos t\, dt$

$= \left[\sin t\right]_0^{\pi/2} - \left[\sin t\right]_{\pi/2}^{3\pi/2} + \left[\sin t\right]_{3\pi/2}^{5\pi/2} - \left[\sin t\right]_{5\pi/2}^{3\pi} = 1 - (-2) + 2 - (-1) = 6$

**101.** $x(t) = t^3 - 6t^2 + 9t - 2$

$x'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t - 3)(t - 1)$

Total distance $= \int_0^5 \left|x'(t)\right| dt$

$= \int_0^5 3\left|(t - 3)(t - 1)\right| dt$

$= 3\int_0^1 (t^2 - 4t + 3)\, dt - 3\int_1^3 (t^2 - 4t + 3)\, dt + 3\int_3^5 (t^2 - 4t + 3)\, dt = 4 + 4 + 20 = 28$ units

**102.** $x(t) = (t - 1)(t - 3)^2 = t^3 - 7t^2 + 15t - 9$

$x'(t) = 3t^2 - 14t + 15$

Using a graphing utility,

Total distance $= \int_0^5 \left|x'(t)\right| dt \approx 27.37$ units.

**103.** Let $c(t)$ be the amount of water that is flowing out of the tank. Then $c'(t) = 500 - 5t$ L/min is the rate of flow.

$$\int_0^{18} c'(t) dt = \int_0^{18} (500 - 5t)\, dt = \left[500t - \frac{5t^2}{2}\right]_0^{18} = 9000 - 810 = 8190 \text{ L}$$

**104.** Let $c(t)$ be the amount of oil leaking and $t = 0$ represent 1 P.M. Then $c'(t) = 4 + 0.75t$ gal/min is the rate of flow.

(a) From 1 P.M. to 4 P.M. (3 hours):

$$\int_0^3 (4 + 0.75t)\, dt = \left[4t + \frac{0.75}{2}t^2\right]_0^3 = \frac{123}{8} = 15.375 \text{ gal}$$

(b) From 4 P.M. to 7 P.M. (3 hours)

$$\int_3^6 (4 + 0.75t)\, dt = \left[4t + \frac{0.75}{2}t^2\right]_3^6 = 22.125 \text{ gal}$$

(c) The second answer is larger because the rate of flow is increasing.

**105.** The function $f(x) = x^{-2}$ is not continuous on $[-1, 1]$.

$$\int_{-1}^1 x^{-2}\, dx = \int_{-1}^0 x^{-2}\, dx + \int_0^1 x^{-2}\, dx$$

Each of these integrals is infinite. $f(x) = x^{-2}$ has a nonremovable discontinuity at $x = 0$.

**106.** The function $f(x) = \dfrac{2}{x^3}$ is not continuous on $[-2, 1]$.

$$\int_{-2}^1 \frac{2}{x^3}\, dx = \int_{-2}^0 \frac{2}{x^3}\, dx + \int_0^1 \frac{2}{x^3}\, dx$$

Each of these integrals is infinite. $f(x) = \dfrac{2}{x^3}$ has a nonremovable discontinuity at $x = 0$.

107. The function $f(x) = \sec^2 x$ is not continuous on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

$$\int_{\pi/4}^{3\pi/4} \sec^2 x\,dx = \int_{\pi/4}^{\pi/2} \sec^2 x\,dx + \int_{\pi/2}^{3\pi/4} \sec^2 x\,dx$$

Each of these integrals is infinite. $f(x) = \sec^2 x$ has a nonremovable discontinuity at $x = \frac{\pi}{2}$

108. The function $f(x) = \csc x \cot x$ is not continuous on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

$$\int_{\pi/2}^{3\pi/2} \csc x \cot x\,dx = \int_{\pi/2}^{\pi} \csc x \cot x\,dx + \int_{\pi}^{3\pi/2} \csc x \cot x\,dx$$

Each of these integrals is infinite $f(x) = \csc x \cot x$ has a nonremovable discontinuity at $x = \pi$.

109. $P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\,d\theta = \left[-\frac{2}{\pi}\cos\theta\right]_0^{\pi/2} = -\frac{2}{\pi}(0 - 1) = \frac{2}{\pi} \approx 63.7\%$

110. Let $F(t)$ be an antiderivative of $f(t)$. Then,

$$\int_{u(x)}^{v(x)} f(t)\,dt = \left[F(t)\right]_{u(x)}^{v(x)} = F(v(x)) - F(u(x))$$

$$\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)\,dt\right] = \frac{d}{dx}\left[F(v(x)) - F(u(x))\right] = F'(v(x))v'(x) - F'(u(x))u'(x) = f(v(x))v'(x) - f(u(x))u'(x).$$

111. True

112. True

113. $f(x) = \int_0^{1/x} \frac{1}{t^2 + 1}\,dt + \int_0^{x} \frac{1}{t^2 + 1}\,dt$

By the Second Fundamental Theorem of Calculus, you have $f'(x) = \frac{1}{(1/x)^2 + 1}\left(-\frac{1}{x^2}\right) + \frac{1}{x^2 + 1} = -\frac{1}{1 + x^2} + \frac{1}{x^2 + 1} = 0.$

Because $f'(x) = 0$, $f(x)$ must be constant.

114. $\int_c^x f(t)\,dt = x^2 + x - 2$

Let $f(t) = 2t + 1$. Then

$$\int_c^x f(t)dt = \int_c^x (2t + 1)dt = \left[t^2 + t\right]_c^x =$$

$$x^2 + x - c^2 - c = x^2 + x - 2$$

$$-c^2 - c = -2$$

$$c^2 + c - 2 = 0$$

$$(c + 2)(c - 1) = 0 \Rightarrow c = 1, -2.$$

So, $f(x) = 2x + 1$, and $c = 1$ or $c = -2$.

115. $G(x) = \int_0^x \left[s\int_0^s f(t)\,dt\right] ds$

(a) $G(0) = \int_0^0 \left[s\int_0^s f(t)\,dt\right] ds = 0$

(b) Let $F(s) = s\int_0^s f(t)\,dt$.

$$G(x) = \int_0^x F(s)\,ds$$

$$G'(x) = F(x) = x\int_0^x f(t)\,dt$$

$$G'(0) = 0\int_0^0 f(t)\,dt = 0$$

(c) $G''(x) = x \cdot f(x) + \int_0^x f(t)\,dt$

(d) $G''(0) = 0 \cdot f(0) + \int_0^0 f(t)\,dt = 0$