4.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Graphical Reasoning **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

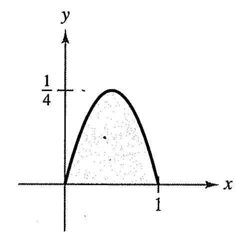
1. $\int_0^{\pi} \frac{4}{x^2+1}\,dx$
2. $\int_0^{\pi} \cos x\,dx$
3. $\int_{-2}^{2} x\sqrt{x^2+1}\,dx$
4. $\int_{-2}^{2} x\sqrt{2-x}\,dx$

Evaluating a Definite Integral **In Exercises 5–34, evaluate the definite integral. Use a graphing utility to verify your result.**

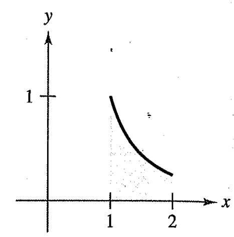
5. $\int_0^{2} 6x\,dx$
6. $\int_{-3}^{1} 8\,dt$
7. $\int_{-1}^{0} (2x-1)\,dx$
8. $\int_{-1}^{2} (7-3t)\,dt$
9. $\int_{-1}^{1} (t^2-2)\,dt$
10. $\int_{1}^{2} (6x^2-3x)\,dx$
11. $\int_{0}^{1} (2t-1)^2\,dt$
12. $\int_{1}^{3} (4x^3-3x^2)\,dx$
13. $\int_{1}^{2} \left(\frac{3}{x^2}-1\right)dx$
14. $\int_{-2}^{-1} \left(u-\frac{1}{u^2}\right)du$
15. $\int_{1}^{4} \frac{u-2}{\sqrt{u}}\,du$
16. $\int_{-8}^{8} x^{1/3}\,dx$
17. $\int_{-1}^{1} (\sqrt[3]{t}-2)\,dt$
18. $\int_{1}^{8} \sqrt{\frac{2}{x}}\,dx$
19. $\int_{0}^{1} \frac{x-\sqrt{x}}{3}\,dx$
20. $\int_{0}^{2} (2-t)\sqrt{t}\,dt$
21. $\int_{-1}^{0} (t^{1/3}-t^{2/3})\,dt$
22. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$
23. $\int_{0}^{5} |2x-5|\,dx$
24. $\int_{1}^{4} (3-|x-3|)\,dx$
25. $\int_{0}^{4} |x^2-9|\,dx$
26. $\int_{0}^{4} |x^2-4x+3|\,dx$
27. $\int_{0}^{\pi} (1+\sin x)\,dx$
28. $\int_{0}^{\pi} (2+\cos x)\,dx$
29. $\int_{0}^{\pi/4} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$
30. $\int_{0}^{\pi/4} \frac{\sec^2\theta}{\tan^2\theta+1}\,d\theta$
31. $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$
32. $\int_{\pi/4}^{\pi/2} (2-\csc^2 x)\,dx$
33. $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$
34. $\int_{-\pi/2}^{\pi/2} (2t+\cos t)\,dt$

Finding the Area of a Region **In Exercises 35–38, determine the area of the given region.**

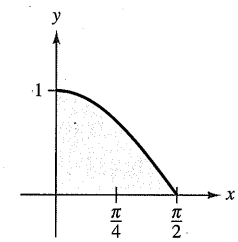
35. $y = x - x^2$



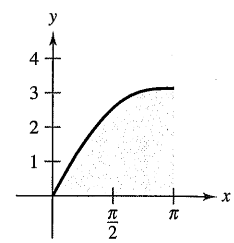
36. $y = \frac{1}{x^2}$



37. $y = \cos x$



38. $y = x + \sin x$



Finding the Area of a Region **In Exercises 39–44, find the area of the region bounded by the graphs of the equations.**

39. $y = 5x^2 + 2,\quad x = 0,\quad x = 2,\quad y = 0$
40. $y = x^3 + x,\quad x = 2,\quad y = 0$
41. $y = 1 + \sqrt[3]{x},\quad x = 0,\quad x = 8,\quad y = 0$
42. $y = 2\sqrt{x} - x,\quad y = 0$
43. $y = -x^2 + 4x,\quad y = 0$
44. $y = 1 - x^4,\quad y = 0$

Using the Mean Value Theorem for Integrals **In Exercises 45–50, find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.**

45. $f(x) = x^3,\quad [0, 3]$
46. $f(x) = \sqrt{x},\quad [4, 9]$
47. $y = \frac{x^2}{4},\quad [0, 6]$
48. $f(x) = \frac{9}{x^3},\quad [1, 3]$
49. $f(x) = 2\sec^2 x,\quad \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
50. $f(x) = \cos x,\quad \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

Finding the Average Value of a Function **In Exercises 51–56, find the average value of the function over the given interval and all values of x in the interval for which the function equals its average value.**

51. $f(x) = 9 - x^2,\quad [-3, 3]$
52. $f(x) = \frac{4(x^2+1)}{x^2},\quad [1, 3]$
53. $f(x) = x^3,\quad [0, 1]$
54. $f(x) = 4x^3 - 3x^2,\quad [0, 1]$
55. $f(x) = \sin x,\quad [0, \pi]$
56. $f(x) = \cos x,\quad \left[0, \frac{\pi}{2}\right]$

57. Velocity The graph shows the velocity, in feet per second, of a car accelerating from rest. Use the graph to estimate the distance the car travels in 8 seconds.

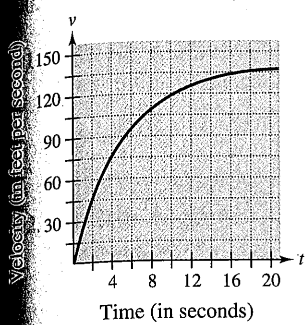


Figure for 57

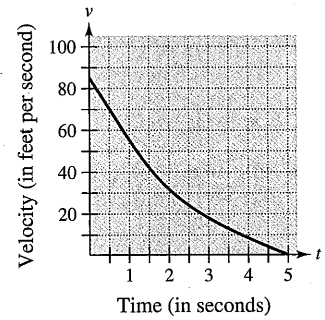
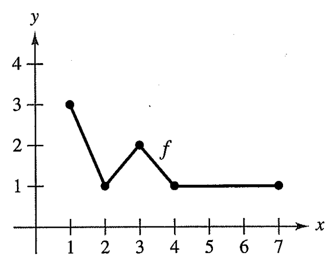


Figure for 58

58. Velocity The graph shows the velocity, in feet per second, of a decelerating car after the driver applies the brakes. Use the graph to estimate how far the car travels before it comes to a stop.

WRITING ABOUT CONCEPTS

59. Using a Graph The graph of f is shown in the figure.



(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) when the graph is translated two units upward.

60. Rate of Growth Let $r'(t)$ represent the rate of growth of a dog, in pounds per year. What does $r(t)$ represent? What does $\int_2^6 r'(t)\,dt$ represent about the dog?

61. Force The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $\sec x$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $[0, \pi/3]$, and $F(0) = 500$.

(a) Find F as a function of x.

(b) Find the average force exerted by the press over the interval $[0, \pi/3]$.

62. Blood Flow The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is

$$v = k(R^2 - r^2)$$

where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

63. Respiratory Cycle The volume V, in liters, of air in the lungs during a five-second respiratory cycle is approximated by the model $V = 0.1729t + 0.1522t^2 - 0.0374t^3$, where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

64. Average Sales A company fits a model to the monthly sales data for a seasonal product. The model is

$$S(t) = \frac{t}{4} + 1.8 + 0.5\sin\left(\frac{\pi t}{6}\right), \quad 0 \le t \le 24$$

where S is sales (in thousands) and t is time in months.

(a) Use a graphing utility to graph $f(t) = 0.5\sin(\pi t/6)$ for $0 \le t \le 24$. Use the graph to explain why the average value of $f(t)$ is 0 over the interval.

(b) Use a graphing utility to graph $S(t)$ and the line $g(t) = t/4 + 1.8$ in the same viewing window. Use the graph and the result of part (a) to explain why g is called the *trend line*.

65. Modeling Data An experimental vehicle is tested on a straight track. It starts from rest, and its velocity v (in meters per second) is recorded every 10 seconds for 1 minute (see table).

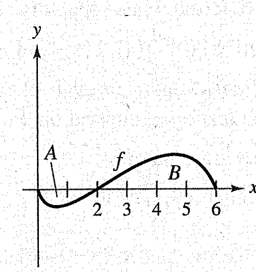
t	0	10	20	30	40	50	60
v	0	5	21	40	62	78	83

(a) Use a graphing utility to find a model of the form $v = at^3 + bt^2 + ct + d$ for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use the Fundamental Theorem of Calculus to approximate the distance traveled by the vehicle during the test.

66. HOW DO YOU SEE IT? The graph of f is shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.



(a) $\int_0^2 f(x)\,dx =$ ▢ (b) $\int_2^6 f(x)\,dx =$ ▢

(c) $\int_0^6 |f(x)|\,dx =$ ▢ (d) $\int_0^2 -2f(x)\,dx =$ ▢

(e) $\int_0^6 [2 + f(x)]\,dx =$ ▢

(f) The average value of f over the interval $[0, 6]$ is ▢.

Evaluating a Definite Integral **In Exercises 67–72, find F as a function of x and evaluate it at $x = 2$, $x = 5$, and $x = 8$.**

67. $F(x) = \int_0^x (4t - 7)\, dt$

68. $F(x) = \int_2^x (t^3 + 2t - 2)\, dt$

69. $F(x) = \int_1^x \frac{20}{v^2}\, dv$

70. $F(x) = \int_2^x -\frac{2}{t^3}\, dt$

71. $F(x) = \int_1^x \cos \theta\, d\theta$

72. $F(x) = \int_0^x \sin \theta\, d\theta$

73. Analyzing a Function Let

$$g(x) = \int_0^x f(t)\, dt$$

where f is the function whose graph is shown in the figure.

(a) Estimate $g(0)$, $g(2)$, $g(4)$, $g(6)$, and $g(8)$.

(b) Find the largest open interval on which g is increasing. Find the largest open interval on which g is decreasing.

(c) Identify any extrema of g.

(d) Sketch a rough graph of g.

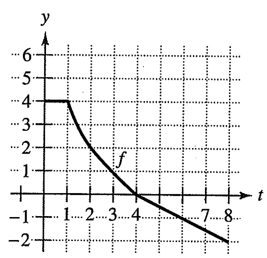


Figure for 73

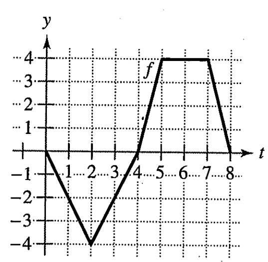


Figure for 74

74. Analyzing a Function Let

$$g(x) = \int_0^x f(t)\, dt$$

where f is the function whose graph is shown in the figure.

(a) Estimate $g(0)$, $g(2)$, $g(4)$, $g(6)$, and $g(8)$.

(b) Find the largest open interval on which g is increasing. Find the largest open interval on which g is decreasing.

(c) Identify any extrema of g.

(d) Sketch a rough graph of g.

Finding and Checking an Integral **In Exercises 75–80, (a) integrate to find F as a function of x, and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).**

75. $F(x) = \int_0^x (t + 2)\, dt$

76. $F(x) = \int_0^x t(t^2 + 1)\, dt$

77. $F(x) = \int_8^x \sqrt[3]{t}\, dt$

78. $F(x) = \int_4^x \sqrt{t}\, dt$

79. $F(x) = \int_{\pi/4}^x \sec^2 t\, dt$

80. $F(x) = \int_{\pi/3}^x \sec t \tan t\, dt$

Using the Second Fundamental Theorem of Calculus **In Exercises 81–86, use the Second Fundamental Theorem of Calculus to find $F'(x)$.**

81. $F(x) = \int_{-2}^x (t^2 - 2t)\, dt$

82. $F(x) = \int_1^x \frac{t^2}{t^2 + 1}\, dt$

83. $F(x) = \int_{-1}^x \sqrt{t^4 + 1}\, dt$

84. $F(x) = \int_1^x \sqrt[4]{t}\, dt$

85. $F(x) = \int_0^x t \cos t\, dt$

86. $F(x) = \int_0^x \sec^3 t\, dt$

Finding a Derivative **In Exercises 87–92, find $F'(x)$.**

87. $F(x) = \int_x^{x+2} (4t + 1)\, dt$

88. $F(x) = \int_{-x}^x t^3\, dt$

89. $F(x) = \int_0^{\sin x} \sqrt{t}\, dt$

90. $F(x) = \int_2^{x^2} \frac{1}{t^3}\, dt$

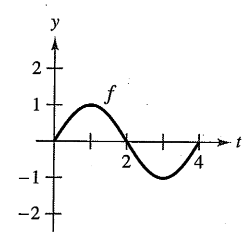
91. $F(x) = \int_0^{x^3} \sin t^2\, dt$

92. $F(x) = \int_0^{x^2} \sin \theta^2\, d\theta$

93. Graphical Analysis Sketch an approximate graph of g on the interval $0 \le x \le 4$, where

$$g(x) = \int_0^x f(t)\, dt.$$

Identify the x-coordinate of an extremum of g. To print an enlarged copy of the graph, go to *MathGraphs.com*



94. Area The area A between the graph of the function

$$g(t) = 4 - \frac{4}{t^2}$$

and the t-axis over the interval $[1, x]$ is

$$A(x) = \int_1^x \left(4 - \frac{4}{t^2}\right) dt.$$

(a) Find the horizontal asymptote of the graph of g.

(b) Integrate to find A as a function of x. Does the graph of A have a horizontal asymptote? Explain.

Particle Motion **In Exercises 95–100, the velocity function, in feet per second, is given for a particle moving along a straight line. Find (a) the displacement and (b) the total distance that the particle travels over the given interval.**

95. $v(t) = 5t - 7, \quad 0 \le t \le 3$

96. $v(t) = t^2 - t - 12, \quad 1 \le t \le 5$

97. $v(t) = t^3 - 10t^2 + 27t - 18, \quad 1 \le t \le 7$

98. $v(t) = t^3 - 8t^2 + 15t, \quad 0 \le t \le 5$

99. $v(t) = \dfrac{1}{\sqrt{t}}, \quad 1 \le t \le 4$

100. $v(t) = \cos t, \quad 0 \le t \le 3\pi$

101. Particle Motion A particle is moving along the x-axis. The position of the particle at time t is given by

$$x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5.$$

Find the total distance the particle travels in 5 units of time.

102. Particle Motion Repeat Exercise 101 for the position function given by

$$x(t) = (t - 1)(t - 3)^2, \quad 0 \le t \le 5.$$

103. Water Flow Water flows from a storage tank at a rate of $(500 - 5t)$ liters per minute. Find the amount of water that flows out of the tank during the first 18 minutes.

104. Oil Leak At 1:00 P.M., oil begins leaking from a tank at a rate of $(4 + 0.75t)$ gallons per hour.

(a) How much oil is lost from 1:00 P.M. to 4:00 P.M.?

(b) How much oil is lost from 4:00 P.M. to 7:00 P.M.?

(c) Compare your answers to parts (a) and (b). What do you notice?

Error Analysis In Exercises 105–108, describe why the statement is incorrect.

105. $$\int_{-1}^{1} x^{-2}\,dx = \left[-x^{-1}\right]_{-1}^{1} = (-1) - 1 = -2$$

106. $$\int_{-2}^{1} \frac{2}{x^3}\,dx = \left[-\frac{1}{x^2}\right]_{-2}^{1} = -\frac{3}{4}$$

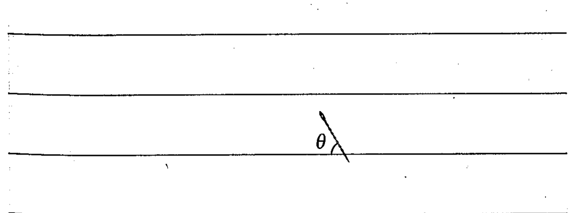
107. $$\int_{\pi/4}^{3\pi/4} \sec^2 x\,dx = \left[\tan x\right]_{\pi/4}^{3\pi/4} = -2$$

108. $$\int_{\pi/2}^{3\pi/2} \csc x \cot x\,dx = \left[-\csc x\right]_{\pi/2}^{3\pi/2} = 2$$

109. Buffon's Needle Experiment A horizontal plane is ruled with parallel lines 2 inches apart. A two-inch needle is tossed randomly onto the plane. The probability that the needle will touch a line is

$$P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\,d\theta$$

where θ is the acute angle between the needle and any one of the parallel lines. Find this probability.



110. Proof Prove that

$$\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)\,dt\right] = f(v(x))v'(x) - f(u(x))u'(x).$$

True or False? In Exercises 111 and 112, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

111. If $F'(x) = G'(x)$ on the interval $[a, b]$, then

$$F(b) - F(a) = G(b) - G(a).$$

112. If f is continuous on $[a, b]$, then f is integrable on $[a, b]$.

113. Analyzing a Function Show that the function

$$f(x) = \int_0^{1/x} \frac{1}{t^2 + 1}\,dt + \int_0^{x} \frac{1}{t^2 + 1}\,dt$$

is constant for $x > 0$.

114. Finding a Function Find the function $f(x)$ and all values of c such that

$$\int_c^x f(t)\,dt = x^2 + x - 2.$$

115. Finding Values Let

$$G(x) = \int_0^x \left[s\int_0^s f(t)\,dt\right] ds$$

where f is continuous for all real t. Find (a) $G(0)$, (b) $G'(0)$, (c) $G''(x)$, and (d) $G''(0)$.

SECTION PROJECT

Demonstrating the Fundamental Theorem

Use a graphing utility to graph the function

$$y_1 = \sin^2 t$$

on the interval $0 \le t \le \pi$. Let $F(x)$ be the following function of x.

$$F(x) = \int_0^x \sin^2 t\,dt$$

(a) Complete the table. Explain why the values of F are increasing.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$F(x)$							

(b) Use the integration capabilities of a graphing utility to graph F.

(c) Use the differentiation capabilities of a graphing utility to graph $F'(x)$. How is this graph related to the graph in part (b)?

(d) Verify that the derivative of

$$y = \frac{1}{2}t - \frac{1}{4}\sin 2t$$

is $\sin^2 t$. Graph y and write a short paragraph about how this graph is related to those in parts (b) and (c).