Practice Problem:

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

#47) $f(x) = 4 - x^2$ $[-2, 2]$ a) Find avg. value
b) find c-value

$$f(c) = \frac{1}{2-(-2)}\int_{-2}^{2} 4 - x^2\,dx$$

$$f(c) = \frac{1}{4}\int_{-2}^{2} 4 - x^2\,dx$$

$$f(c) = \frac{1}{4}\cdot\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 4(2) - \frac{2^3}{3} - \left(4(-2) - \frac{(-2)^3}{3}\right)$$

$$\frac{1}{4}\left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right] = \frac{1}{4}\left[16 - \frac{16}{3}\right] = \frac{1}{4}\left(\frac{32}{3}\right) = \frac{8}{3}$$

a) $\boxed{f(c) = 8/3}$

b) $f(x) = 4 - x^2$

$\frac{8}{3} = 4 - x^2$

$x^2 = 4 - 8/3$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}}$

$\boxed{c = \frac{2}{\sqrt{3}},\ c = \frac{-2}{\sqrt{3}} \text{ in } [-2, 2]}$

Ex. 2 Find $\frac{d}{dx}\left[\int_{2x^3}^{5} \frac{2t}{5-t^2}\,dt\right]$

$$* \quad \frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$$

$$\frac{d}{dx}\int_5^{2x^3} \frac{-2t}{5-t^2}\,dt$$

$$= \frac{-2(2x^3)}{5-(2x^3)^2}\cdot 6x^2$$

$$= \boxed{\frac{-24x^5}{5-4x^6}}$$