2017

$\boxed{4.4b}$ Avg. Value Theorem, SFTC p. 288–289

#35–55 odd, 75–91 odd

45) MVT for Integrals (Avg. value theorem) * $\frac{1}{b-a}\int_a^b f(x)\,dx$

$f(x) = x^3 \quad [0,3]$

$$\frac{1}{3-0}\int_0^3 x^3\,dx = \frac{1}{3}\cdot\frac{x^4}{4}\Big]_0^3 = \frac{1}{3}\left(\frac{3^4}{4}\right) = \frac{27}{4}$$

$$f(c) = \frac{27}{4} \to x^3 = \frac{27}{4} \quad x = \sqrt[3]{\frac{27}{4}} \to \boxed{c = \frac{3}{\sqrt[3]{4}} \approx 1.889 \text{ in } [0,3]}$$

49) $f(x) = \sec^2 x \quad \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (a, b)

$$\frac{1}{\pi/4 - (-\pi/4)}\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{1}{2\pi/4}\cdot \tan x\Big]_{-\pi/4}^{\pi/4} = \frac{2}{\pi}\tan\pi/4 - \frac{2}{\pi}\tan(-\pi/4)$$

$$= \frac{2}{\pi}(1) - \frac{2}{\pi}(-1) = \frac{4}{\pi}$$

$\sec^2 x = \frac{4}{\pi}$

$\sec x = \frac{2}{\sqrt{\pi}}$

$x = \pm\sec^{-1}\left(\frac{2}{\sqrt{\pi}}\right)$

$c = \pm\cos^{-1}\left(\frac{\sqrt{\pi}}{2}\right) \approx \boxed{\pm 0.4817}$

55) Avg. value of function: $f(x) = \sin x \quad [0, \pi]$

$$\frac{1}{\pi - 0}\int_0^\pi \sin x\,dx = \frac{1}{\pi}\cdot -\cos x\Big]_0^\pi = \frac{1}{\pi}(-\cos\pi) - \frac{1}{\pi}(-\cos 0)$$

$$= \frac{1}{\pi}(1) + \frac{1}{\pi}(1) = \frac{2}{\pi}$$

$\sin x = \frac{2}{\pi}$

$x = \sin^{-1}\left(\frac{2}{\pi}\right)$

$\boxed{c \approx 0.690, 2.451}$

$\boxed{4.4b}$ #75-91 SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

Apply 2nd Fundamental Theorem of Calculus

81) $F(x) = \int_{-2}^{x} t^2-2t\,dt \quad F'(x) = \frac{d}{dx}\int_{-2}^{x} t^2-2t\,dt = \boxed{x^2-2x}$

85) $F(x) = \int_0^x t\cos t\,dt \quad F'(x) = \frac{d}{dx}\int_0^x t\cos t\,dt = \boxed{x\cos x}$

87) $F(x) = \int_x^{x+2} (4t+1)\,dt \quad * \frac{d}{dx}\int_{g(x)}^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x) - f(g(x))\cdot g'(x)$

$F'(x) = \frac{d}{dx}\int_x^{x+2} 4t+1\,dt = [4(x+2)+1]\cdot 1 - [4x+1]\cdot 1$

$= 4x+8+1-4x-1 = \boxed{8}$

89) $F(x) = \int_0^{\sin x} \sqrt{t}\,dt \quad F'(x) = \frac{d}{dx}\int_0^{\sin x} \sqrt{t}\,dt = \boxed{\sqrt{\sin x}\cdot\cos x}$

91) $F(x) = \int_0^{x^3} \sin t^2\,dt \quad F'(x) = \frac{d}{dx}\int_0^{x^3} \sin t^2\,dt = \sin(x^3)^2\cdot 3x^2$

$= \boxed{3x^2\sin(x^6)}$