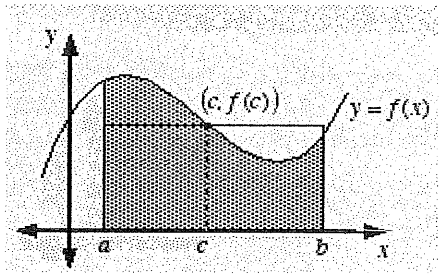
If function f is integrable on the closed interval [a, b], then the average value of f on the interval is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$



*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

Example 1: a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

2nd Fundamental Theorem of Calculus (SFTC)

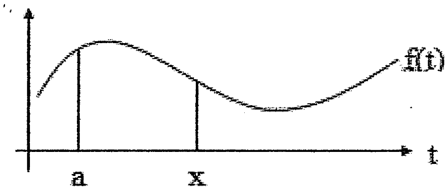
Definite Integral as a Function

To recap, we've covered:

1) Indefinite General Integrals (Area-finding functions)
2) Definite Integrals (Finds Area between 2 x-values)

There is also now a function that is the integral itself. Instead of going from a constant to another constant, we are going from a constant to a moving value of x.

Consider: $f(x)=\int_a^x f(t)dt$



2nd Fundamental Theorem of Calculus **Very Important**

Applies the concept that derivative and integrals are inverse operations of each other.

1) $\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)$ (a is a constant)

2) $\frac{d}{dx}\left[\int_{q(x)}^{p(x)} f(t)dt\right]=f(p(x))\cdot p'(x)-f(q(x))\cdot q'(x)$

Example 2:

a) $\frac{d}{dx}\left[\int_{-3}^{x}\sqrt{t^2+4}dt\right]=$

b) $\frac{d}{dx}\left[\int_{3}^{x^2}\sqrt{t-1}dt\right]=$

c) $\frac{d}{dx}\left[\int_{10}^{x^2}\sqrt{t-1}dt\right]=$

d) $\frac{d}{dx}\left[\int_{3x}^{0}\frac{1}{t+2}dt\right]=$

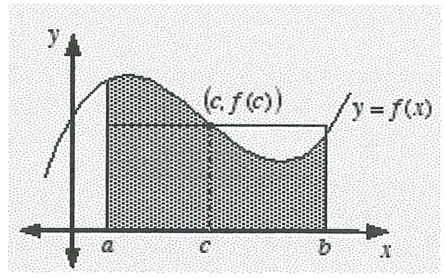
e) $\frac{d}{dx}\left[\int_{x}^{x^2}(2t+3)dt\right]=$

Key

Calculus **Chapter 4.4b** **Average Value Theorem (MVT for Integrals)**

If function $\boldsymbol{f}$ is integrable on the closed interval [a, b], then the average value of $\boldsymbol{f}$ on the interval is

$$\underbrace{f(c)}_{\text{height}}=\frac{1}{\underbrace{b-a}_{\text{width}}}\underbrace{\int_a^b f(x)dx}_{\text{Area}}$$



Since height • width = Area, $\text{height}=\frac{\text{Area}}{\text{width}}$

*There exists a rectangle such that the area of the rectangle is the same as the area under the curve (shaded region). f (c) is the height of the rectangle

Example 1: a) Find the average value of $f(x) = x^2 + 1$ on [2, 5]. b) find the c value

* Use Avg. Value Theorem: $f(c)=\frac{1}{5-2}\int_2^5 x^2+1\,dx$

a) $f(c)=\frac{1}{3}\cdot\frac{x^3}{3}+x\Big]_2^5=\frac{1}{3}\left[\frac{5^3}{3}+5-\left(\frac{2^3}{3}+2\right)\right]$

$=\frac{1}{3}\left[\frac{117}{3}+3\right]=\frac{1}{3}\left(\frac{126}{3}\right)=14$ $\boxed{f(c)=14}$

b) Find c-value

$f(x)=x^2+1$

$f(c)=c^2+1$

$14=c^2+1$

$13=c^2$

$c=\pm\sqrt{13}$

$\boxed{c=\sqrt{13} \text{ since } 2<\sqrt{13}<5}$

2nd Fundamental Theorem of Calculus (SFTC)

Definite Integral as a Function

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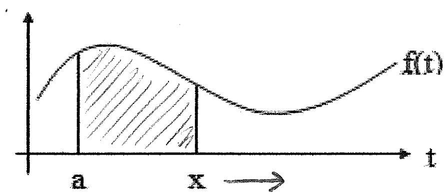

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1) $\frac{d}{dx}\left[\int_a^{p(x)} f(t)dt\right] = f(p(x)) \cdot p'(x)$ (a is a constant)

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Example 2:

a) $\frac{d}{dx}\left[\int_{-3}^{x} \sqrt{t^2+4}dt\right] = \sqrt{x^2+4} \cdot 1$

$= \boxed{\sqrt{x^2+4}}$

b) $\frac{d}{dx}\left[\int_{3}^{x^2} \sqrt{t-1}dt\right] = \sqrt{x^2-1} \cdot 2x$

$= \boxed{2x\sqrt{x^2-1}}$

c) $\frac{d}{dx}\left[\int_{10}^{x^2} \sqrt{t-1}dt\right] = \sqrt{x^2-1} \cdot 2x$

$= \boxed{2x\sqrt{x^2-1}}$

d) $\frac{d}{dx}\left[\int_{3x}^{0} \frac{1}{t+2}dt\right] =$

$\frac{d}{dx}\left[-\int_0^{3x} \frac{1}{t+2}dt\right] = -\frac{1}{3x+2} \cdot 3 = \boxed{\frac{-3}{3x+2}}$

e) $\frac{d}{dx}\left[\int_{x}^{x^2} (2t+3)dt\right] = \left[2(x^2)+3\right] \cdot (2x) - (2x+3)(1)$

$= 4x^3 + 6x - 2x - 3$

$= \boxed{4x^3 + 4x - 3}$